Chain Rule (brackets)

Starter

1. (Review of last lesson) Find the area between the x−axis and the curve $y = \cos x$ between $x = 0$ and $x = \frac{2\pi}{3}$.
   Hint: Draw a sketch to help.

2. Find $\frac{dy}{dx}$ when: (a) $y = (3x - 4)^2$ (b) $y = (3x - 4)^{17}$.
   N.B. Don't spend too long on (b) if you are having difficulty…

Notes

To differentiate $y = (3x - 4)^{17}$, and similar functions involving special functions, we "Let $u = \ldots$" and then use the Chain Rule.

Chain Rule: $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

After we have done a few examples, we should spot a pattern and be able to differentiate $y = (3x - 4)^{17}$ in our heads.

Differentiating brackets raised to a power

We can differentiate $3x - 4$ and x^{17} easily so the chain rule gets us to do them separately.

E.g. Differentiate $y = (3x - 4)^{17}$.

Working: Let $u = 3x - 4$ so $\frac{du}{dx} = 3$

$y = u^{17}$ so $\frac{dy}{du} = 17u^{16}$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 3 \times 17u^{16}$

Replace u by $3x - 4$ and simplify: $\frac{dy}{dx} = 51(3x - 4)^{16}$

Here's another example.

E.g. Differentiate $y = (7 - 4x)^6$.

Working: Let $u = 7 - 4x$ so $\frac{du}{dx} = -4$

$y = u^6$ so $\frac{dy}{du} = 6u^5$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = -4 \times 6u^5$

Replace u by $7 - 4x$ and simplify: $\frac{dy}{dx} = -24(7 - 4x)^5$

E.g. 1 Differentiate: (a) $y = (5x + 2)^8$ (b) $y = (x^2 + 3x - 9)^5$

(c) $f(x) = \sin^2 x$ (d) $y = (ax + b)^n$

Hint: For (c), rewrite $f(x) = \sin^2 x$ as $f(x) = (\sin x)^2$.

You might already have spotted the pattern but let's look at the general case.

E.g. 2 Differentiate $y = \left[f(x)\right]^n$

Working: Let $u = f(x)$ so $\dfrac{du}{dx} = f'(x)$

$y = u^n$ so $\dfrac{dy}{du} = nu^{n-1}$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = f'(x) \times nu^{n-1}$$

Replace u by $f(x)$: $\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$

The power is reduced by 1

The power comes down to the front

Multiply by the derivative of the bracket

In general: $y = k\left[f(x)\right]^n \Rightarrow \dfrac{dy}{dx} = knf'(x)\left[f(x)\right]^{n-1}$

With practice you will be able to remember the formula and be able to go straight to the answer without using "Let $u = \ldots$".

E.g. Differentiate $y = (x^2 - 7)^4$.

Working: $\dfrac{dy}{dx} = 4 \times 2x \times (x^2 - 9)^3 = 8x(x^2 - 9)^3$

$n \quad f'(x) \quad \left[f(x)\right]^{n-1}$

E.g. 3 Without using "Let $u = \ldots$", differentiate $y = 6(x^3 - 5x)^{11}$

Video: Chain rule (brackets)

Solutions to Starter and E.g.s

Exercise
p201 10A Qu 1iabc, 2iagh, 3iab

Summary

Chain Rule: $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$

$y = k\left[f(x)\right]^n \Rightarrow \dfrac{dy}{dx} = knf'(x)\left[f(x)\right]^{n-1}$