Chain Rule (trigonometry)

Starter

1. **(Review of last lesson)** Differentiate $y = 5\ln(4x - 3)$.

2. Using the chain rule to differentiate the following:
(a) $y = \cos(x^2 - 5x + 3)$ (b) $y = \tan(6x - 7)$ (c) $y = k\sin\left[f(x)\right]$

Notes

In general:

- $y = k\sin\left[f(x)\right]$ $\dfrac{dy}{dx} = kf'(x)\cos\left[f(x)\right]$
- $y = k\cos\left[f(x)\right]$ $\dfrac{dy}{dx} = -kf'(x)\sin\left[f(x)\right]$
- $y = k\tan\left[f(x)\right]$ $\dfrac{dy}{dx} = kf'(x)\sec^2\left[f(x)\right]$

E.g. 1 Write down the derivatives of:
(a) $f(x) = 7\tan(5x - 2)$ (b) $f(x) = 8\cos(6x^2 - 11x + 1)$

Working: (a) $f'(x) = 7 \times 5 \times \sec^2(5x - 2) = 35\sec^2(5x - 2)$

Sometimes a question will require us to use the chain rule twice

E.g. 2 Find the derivative of $y = \sin^3(7x - 1)$.

Hint: rewrite as : $y = \left[\sin(7x - 1)\right]^3$ so that it is in the form $y = \left[f(x)\right]^n$.
Use the chain rule when differentiating $[\ldots]^3$ and $\sin(7x - 1)$

Remember: $\sec x = \dfrac{1}{\cos x}$ $cosec x = \dfrac{1}{\sin x}$ $\cot x = \dfrac{1}{\tan x}$

E.g. 3 (a) By writing $y = \sec x$ as $y = (\cos x)^{-1}$, find $\dfrac{dy}{dx}$.

(b) Using a similar method, find $\dfrac{dy}{dx}$ when:
(i) $y = cosec x$
(ii) $y = \cot x$

Working: (a) $y = (\cos x)^{-1} \Rightarrow$

$$\frac{dy}{dx} = -1 \times -\sin x \times (\cos x)^{-2}$$
$$= \frac{\sin x}{\cos x^2}$$
$$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$$
$$= \sec x \tan x$$

(b) (i) $y = (\sin x)^{-1} \Rightarrow$

$$\frac{dy}{dx} = -1 \times \cos x \times (\sin x)^{-2}$$
$$= -\frac{\cos x}{\sin x^2}$$
$$= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$$
$$= -cosecx \cot x$$

(ii) $y = (\tan x)^{-1} \Rightarrow$

$$\frac{dy}{dx} = -1 \times \sec^2 x \times (\tan x)^{-2}$$
$$= -\frac{\sec^2 x}{\tan x^2}$$
$$= -\frac{1}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x}$$
$$= -\frac{1}{\sin x^2}$$
$$= -cosec^2 x$$

Video: Chain rule (trigonometry)

Solutions to Starter and E.g.s

Exercise
p201 10A Qu 1ie, 2ide, 3ic
p201 10A Qu 5-13 (not 14 cosec)

Summary

Chain Rule: $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$y = k[f(x)]^n \Rightarrow \frac{dy}{dx} = knf'(x)[f(x)]^{n-1}$

$y = ke^{f(x)} \Rightarrow \frac{dy}{dx} = kf'(x)e^{f(x)}$

$y = k\ln[f(x)] \Rightarrow \frac{dy}{dx} = \frac{kf'(x)}{f(x)}$

$y = k\sin[f(x)] \Rightarrow \frac{dy}{dx} = kf'(x)\cos[f(x)]$

$y = k\cos[f(x)] \Rightarrow \frac{dy}{dx} = -kf'(x)\sin[f(x)]$

$y = k\tan[f(x)] \Rightarrow \frac{dy}{dx} = kf'(x)\sec^2[f(x)]$

$$y = k\sec[f(x)] \quad \Rightarrow \quad \frac{dy}{dx} = kf'(x)\sec[f(x)]\tan[f(x)]$$

$$y = k\,cosec[f(x)] \quad \Rightarrow \quad \frac{dy}{dx} = -kf'(x)\,cosec[f(x)]\cot[f(x)]$$

$$y = k\cot[f(x)] \quad \Rightarrow \quad \frac{dy}{dx} = -kf'(x)\,cosec^2[f(x)]$$