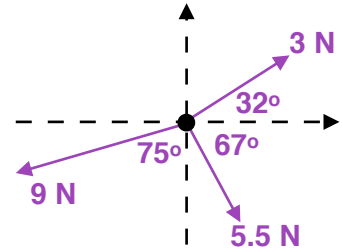


Coefficient of Friction

Starter

- (Review of last lesson) Find the magnitude and direction of the resultant force acting on the particle.



- (Review of last lesson) A trolley is on a level, smooth floor. A girl is trying to push it with a force of 10 N at 15° to the horizontal. Her brother prevents it from moving by pushing horizontally with a force in the opposite direction. With what force does he push?

Notes

- Smooth \Rightarrow no frictional force
 Rough \Rightarrow frictional force present

Key ideas about friction

- Friction acts in the **opposite direction to motion**
- The maximum value of the frictional force is called the **limiting friction** value or F_{lim} .
- By N3L, **as the force pushing** an object **increases** so does the frictional force resisting motion. The **friction force** opposing motion **increases** until it reaches F_{lim} , i.e. the maximum frictional force.
- If an object is **on the point of moving**, the object is said to be in **limiting equilibrium**.
- The limiting frictional force between two surfaces is proportional to the normal contact force. If the limiting frictional is F_{lim} and the normal contact force is R , then $F_{lim} = \mu R$, where μ is a constant. The constant μ is called the **coefficient of friction** and has no units.
- Moving** objects: the frictional force is $F_{lim} = \mu R$.
- Stationary** objects: the frictional force is $F_{lim} \leq \mu R$.
- If contact between two surfaces is smooth, $\mu = 0$.

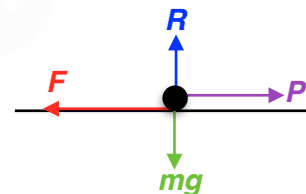
N.B. Experiments indicate that friction when in motion, friction is slightly less than the limiting friction value.

E.g. 1 A horizontal force P newtons is applied to a body of weight 80 N, standing in rough contact with a horizontal plane. The coefficient of friction between the body and the plane is 0.5.

What is the magnitude of the frictional force, F , when:

- $P = 10$ N
- $P = 40$ N
- $P = 50$ N?

State in each case whether or not the body moves.



Solving friction problems

When doing friction questions you can do the following three things:

- $R(\uparrow)$ — resolve vertically to find R , the normal contact force.
- $F_{lim} = \mu R$ — to find the limiting (maximum) frictional force.
- $R(\rightarrow)$ or $F = ma(\rightarrow)$ — resolve or use $F = ma$ horizontally

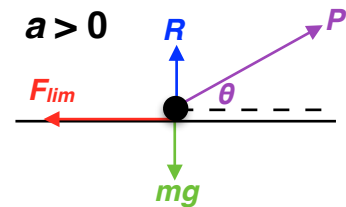
N.B. The difference is just which order you do them in — often this does not matter.

E.g. 2 A particle of mass m kg lies on a rough horizontal plane. The coefficient of friction between the particle and the place is μ . A force of P N acts at an angle θ above the horizontal. For the situations where the particle is:

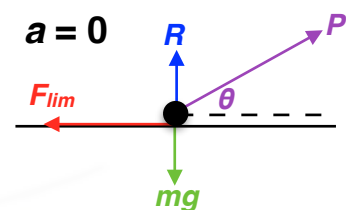
- (a) accelerating and
- (b) in limiting equilibrium or moving with constant speed:
 - (i) Resolve vertically, $R(\uparrow)$
 - (ii) Use $F_{lim} = \mu R$
 - (iii) Use $F = ma(\rightarrow)$ or resolve horizontally $R(\rightarrow)$

Working:

- (a) **Accelerating**
 - (i) $R(\uparrow)$: $R + P \sin \theta = mg$
 $R = mg - P \sin \theta$
 - (ii) $F_{lim} = \mu R$: $F_{lim} = \mu(mg - P \sin \theta)$
 - (iii) $F = ma(\rightarrow)$: $P \cos \theta - F_{lim} = ma$



- (b) **Stationary or moving with constant speed**
 - (i) $R(\uparrow)$: $R + P \sin \theta = mg$
 $R = mg - P \sin \theta$
 - (ii) $F_{lim} = \mu R$: $F_{lim} = \mu(mg - P \sin \theta)$
 - (iii) $R(\rightarrow)$: $P \cos \theta = F_{lim}$



E.g. 3 A small block of weight 32 N is lying in rough contact on a horizontal plane. A horizontal force of P newtons is applied to the block until it is just about to move the block.

- (a) If $P = 8$, find the coefficient of friction, μ , between the block and the plane.
- (b) If $\mu = 0.4$, find the value of P .

E.g. 4 A particle of mass 5 kg, resting on a rough plane, is acted on by a force of 23 N. The coefficient of friction between the particle and surface is $\frac{1}{3}$.

- (a) Find the acceleration when the force acts horizontally.
- (b) If the force acts at an angle of 20° to the horizontal will the acceleration increase or decrease.
- (c) Find the new acceleration.

In general:

Horizontal force:
$$a = \frac{P - \mu mg}{m}$$

Force at an angle θ to the horizontal:
$$a = \frac{P(\sin \theta + \cos \theta) - \mu mg}{m}$$

$$= \frac{P\sqrt{2} \sin(\theta + 45) - \mu mg}{m}$$

So maximum acceleration is when $\theta = 45^\circ$.

Finding the contact force

The contact force between two surfaces is the resultant from the frictional force and the normal contact force.

The magnitude of the contact force, C :
$$C = \sqrt{R^2 + F_{lim}^2}$$

E.g. 5 A particle of mass 12 kg is being pulled along a rough surface by a horizontal force of P N such that its acceleration is 3 m/s^2 . The coefficient of friction between the particle and surface is $\frac{3}{4}$.

- Find the value of P .
- Find the magnitude and direction of the contact force.

[Video: Friction](#)

[Video: Motion on rough horizontal plane](#)

[Motion on rough horizontal plane EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p481 21B Qu 1i, 2-9

Summary

When solving friction problems do the following three things:

- $R(\uparrow)$ – resolve vertically to find R , the normal contact force.
- $F_{lim} = \mu R$ – to find the limiting (maximum) frictional force.
- $R(\rightarrow)$ or $F = ma(\rightarrow)$ – resolve or use $F = ma$ horizontally

The magnitude of the contact force, C :
$$C = \sqrt{R^2 + F_{lim}^2}$$