Composite functions

Starter

1. For the function, $y = \ln(x^2 + x - 12)$, find the largest possible domain and the corresponding range. Write your answers in set notation.

2. Let $f(x) = x^2$ and $g(x) = 5x - 3$. Find:
   (a) $f(-7)$ (b) $g(6)$ (c) $f(x-5)$ (d) $g(2x-9)$

Notes

Composite functions are when the output from one function becomes the input in another, or the same, function.

Notation

$fg(x) \equiv f(g(x)) \equiv fog(x)$ and $ff(x) \equiv f(f(x)) \equiv fof(x) \equiv f^2(x)$

N.B. $f(x) \times f(x) = \left[f(x)\right]^2$

What does $fg(x)$ mean?

$fg(x)$ means do g first and then do f.

Consider composite functions as reading from right to left: $\overleftarrow{fg(x)}$

$fg(5)$ means substitute 5 into $g(x)$ and the output is then substituted into $f(x)$.

$gf(-3)$ means substitute -3 into $f(x)$ and the output is then substituted into $g(x)$.

E.g. Let $f(x) = 3x - 1$ and $g(x) = x^2 - 6$.
(a) Find the value of: (i) $fg(-4)$ (ii) $gf(2)$
(b) Find expressions for: (i) $gf(x)$ (ii) $fg(x)$

Working: (a) (i) $fg(-4)$ $\Rightarrow$ do g first

$g(-4) = (-4)^2 - 6 = 10$

$fg(-4) = f(10) = 3 \times 10 - 1 = 29$

(ii) $gf(2)$ $\Rightarrow$ do f first

$f(2) = 3 \times 2 - 1 = 5$

$gf(2) = g(5) = 5^2 - 6 = 19$

(b) (i) $gf(x)$ substitutes $f(x)$ into $g(x)$

$gf(x) = g(3x-1) = (3x-1)^2 - 6 = 9x^2 - 6x - 5$

(ii) $fg(x)$ substitutes $g(x)$ into $f(x)$

$fg(x) = f(x^2 - 6) = 3(x^2 - 6) - 1 = 3x^2 - 19$

E.g. 1 Let $f(x) = 8x - 15$, $g(x) = x^2 + 1$ and $h(x) = \frac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$ or $x \neq \frac{15}{8}$.

(a) Find the exact value of:

(i) $fg(-3)$ (ii) $gf(3)$ (iii) $fhg(4)$ (iv) $hgf(1)$

(b) Find expressions in terms of x for:

(i) $hf(x)$ (ii) $fh(x)$ (iii) $ff(x)$

(iv) $g^2(x)$ (v) $fgf(x)$ (vi) $hgf(x)$

E.g. 2 Given that $f(x) = ax + b$ and $f(f(x)) = 9x - 28$, find the possible values of a and b.

E.g. 3 The functions $f(x) = 4x + 1$ and $g(x) = ax + b$ are such that $fg = gf$ for $x \in \mathbb{R}$. Find an expression for a in terms of b.

Video: Composite functions

Solutions to Starter and E.g.s

Exercise

p24 2C Qu 1i, 2i, 3i, 4, 5i, 6-8, (9 red)

Summary

Composite functions are when the output from one function becomes the input in another, or the same, function.

$f(g(x)) \equiv fg(x) \equiv fog(x)$ and $f(f(x)) \equiv ff(x) \equiv fof(x) \equiv f^2(x)$

Consider composite functions as reading from right to left: $\overleftarrow{fg}(x)$ do g first and then do f

$fg(5)$ means substitute 5 into $g(x)$ and the output is then substituted into $f(x)$.

$gf(-3)$ means substitute -3 into $f(x)$ and the output is then substituted into $g(x)$.

N.B. $f(x) \times f(x) = [f(x)]^2$