Definite Integration by Substitution

Starter

1. **(Review of last lesson)** Use a suitable substitution to integrate $\int \frac{2x+1}{(x-3)^6} dx$.

Notes

When using integration by substitution with ***definite integration***, the ***limits also need to be changed***.

It is useful to do a little table as part of your working — do this ***before*** any substitution.

x	u

E.g. 1 Use the substitution $u = x + 3$ to find $\int_{-2}^{1} 2x(x+3)^4 dx$.

Working: Let $u = x + 3$ so $x = u - 3$

$$\frac{du}{dx} = 1 \quad \Rightarrow \quad du = dx$$

Table of changed values:

When $x = 1$, $u = 1 + 3 = 4$

When $x = -2$, $u = -2 + 3 = 1$

x	u
1	4
-2	1

$$\int_{-2}^{1} 2x(x+3)^4 dx = \int_{1}^{4} 2(u-3)u^4 du \quad \textit{replace } x+3, dx \textit{ and limits}$$

$$= \int_{1}^{4} 2u^5 - 6u^4 du \quad \textit{expand brackets}$$

$$= \left[\frac{u^6}{3} - \frac{6u^5}{5}\right]_1^4 \quad \textit{integrate with respect to } u$$

$$= \left(\frac{4^6}{3} - \frac{6 \times 4^5}{5}\right) - \left(\frac{1}{3} - \frac{6}{5}\right) \quad \textit{substitute}$$

$$= \frac{687}{5} = 137.4$$

N.B. You can replace the u after integrating and use the original limits but this tends to be more time consuming. In such case leave the limits off the integration symbol while du is written.

E.g. 2 Find: (a) $\int_0^1 \frac{e^x}{(1+e^x)^2}dx$ (b) $\int_1^5 \frac{x}{\sqrt{3x+1}}dx$

Video: Integration by substitution
Video: Integration by substitution involving square roots

Integration by substitution EQ

Solutions to Starter and E.g.s

Exercise
p229 11C Qu 4i, 5-8

Summary
Let $u = \ldots$
Make sure all terms in x and dx are replaced by terms in u and du
Change the limits using a table:

x	u