

## Derivative of $e^x$ and $\ln x$

### Starter

We have already looked at the rate of change of  $e^x$  at AS level.

1. **(Review of AS material)** Copy and complete: the rate of change of  $e^x$  is...
2. **(Review of AS material)** The number of bacteria in a petri dish is given by  $N = e^{5t}$ . Write down the rate of change of  $N$ .
3. Let  $y = 6e^{9x}$ . Find  $\frac{dy}{dx}$ .
4. State the derivatives of: (a)  $e^x$  (b)  $Ae^{kx}$
5. Express  $\ln x^4$  in the form  $k \ln x$ .
6. Simplify: (a)  $e^{\ln x}$  (b)  $\ln(e^x)$  (c)  $e^{7 \ln x}$

### Notes (derivative of $e^x$ )

Function:  $y = Ae^x$  Derivative:  $\frac{dy}{dx} = Ae^x$

Function:  $y = Ae^{kx}$  Derivative:  $\frac{dy}{dx} = Ake^{kx}$

**E.g. 1** Find  $\frac{dy}{dx}$  given that: (a)  $y = 2e^x$  (b)  $y = 5e^{7x}$  (c)  $y = \frac{9}{e^x}$

**Hint:** For (c), transform it so that  $e^x$  is in the numerator.

**E.g. 2** Find the  $x$ -coordinate of the stationary point for the curve  $y = e^{3x} - 6x$  and determine the nature of the point. Leave your answer in exact form.

### Notes (derivative of $\ln x$ )

What is the derivative of the function  $y = \ln x$ ?

Using the Classwiz calculator we can quickly calculate the values of the derivative of  $\ln x$  at various points and try to spot a pattern.

**Video:** [Finding derivatives at a point using a calculator](#)

*There is no need to copy the table.*

x	1	2	3	4	5	8	10
ln x	0	0.693	1.10	1.39	1.61	2.08	2.30
Gradient	1	0.5	0.333	0.25	0.2	0.125	0.1

By inspection, you might be able to see that the derivative of  $\ln x$  is  $\frac{1}{x}$ .

<b>Function:</b>	$y = \ln x$	<b>Derivative:</b>	$\frac{dy}{dx} = \frac{1}{x}$
<b>Function:</b>	$y = A \ln x$	<b>Derivative:</b>	$\frac{dy}{dx} = \frac{A}{x}$

**E.g. 3** Find the derivative of  $y = \ln kx$ .

**Working:** The first time we differentiate  $\ln kx$  we will use laws of logs but after that we can just use the result

$$y = \ln kx = \ln k + \ln x$$

$\ln k$  is just a constant so when we differentiate, it becomes zero

$$\text{So } \frac{dy}{dx} = 0 + \frac{1}{x} = \frac{1}{x}$$

<b>Function:</b>	$y = \ln kx$	<b>Derivative:</b>	$\frac{dy}{dx} = \frac{1}{x}$
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**E.g. 4** Find  $f'(x)$ :

(a)	$f(x) = 8 \ln x$	(b)	$f(x) = \frac{2}{5} \ln x$
(c)	$y = \ln 6x$	(d)	$f(x) = \ln\left(\frac{9}{x}\right)$

**E.g. 5** Find equation of the normal to the curve  $y = \ln 3x$  at the point where  $x = 4$ . Give your answer exactly.

[Video: Derivative of  \$e^x\$](#)

[Video: Derivative of  \$\ln x\$](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p189 9A Qu 1abc, 2abcdfi, 3-6, 9, 15, 17

### Summary

$$y = Ae^{kx} \Rightarrow \frac{dy}{dx} = Ake^{kx}$$

$$y = A \ln kx \Rightarrow \frac{dy}{dx} = \frac{A}{x}$$