Derivatives of Trigonometric Functions

Starter

1. **(Review of last lesson)** Find the equation of the tangent of the curve $y = 8e^x$ at the point where $x = 2$. Give your answer exactly.

Notes

N.B. All calculus with trigonometry must be done in radians, which you have already covered.

$$\text{Degrees} \quad \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \quad \text{Radians}$$

Similar to what we did with $\ln x$ we can find the gradient of each curve at specific points and see if we can spot a pattern. If time permits, a formal proof may be done later in the course.

Video: **Finding derivatives at a point using a calculator**

There is no need to copy this table.

x	0^c	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin x	0	0.5	0.71	0.87	1	0.87	0.71	0.5	0
Gradient	1	0.87	0.71	0.5	0	-0.5	-0.71	-0.87	-1

x	0^c	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
cos x	1	0.87	0.71	0.5	0	-0.5	-0.71	-0.87	-1
Gradient	0	-0.5	-0.71	-0.87	-1	-0.87	-0.71	-0.5	0

By inspection, you might be able to spot the following:

$\frac{d(\sin x)}{dx} = \cos x$ – when we differentiate sine we get cos

$\frac{d(\cos x)}{dx} = -\sin x$ – when we differentiate cos we get negative sine

Here is a result you get for free (we will prove it when we get to the quotient rule in a few week's).

$$\frac{d(\tan x)}{dx} = \frac{1}{\cos^2 x}$$

Soon you will study the reciprocal trigonometric functions and they are included for completeness.

$$cosec x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

So $\dfrac{d(\tan x)}{dx} = \sec^2 x$

In general:
$$y = k\sin x \quad \Rightarrow \quad \frac{dy}{dx} = k\cos x$$
$$y = k\cos x \quad \Rightarrow \quad \frac{dy}{dx} = -k\sin x$$
$$y = k\tan x \quad \Rightarrow \quad \frac{dy}{dx} = k\sec^2 x$$

E.g. 1 Differentiate the following:
(a) $y = 5\sin x$ (b) $f(x) = -8\cos x$ (c) $f(x) = 7\tan x - 2\sin x$

E.g. 2 Find the equation of the tangent to the curve $f(x) = 4\sin x$ where $x = \dfrac{\pi}{3}$. Give your answer exactly.

Video: Derivatives of sin/cos/tan

Solutions to Starter and E.g.s

Exercise
p189 9A Qu 1def, 2e, 7, 8, 10-14, 18

Summary
$$y = Ae^{kx} \quad \Rightarrow \quad \frac{dy}{dx} = Ake^{kx}$$
$$y = A\ln kx \quad \Rightarrow \quad \frac{dy}{dx} = \frac{A}{x}$$
$$y = k\sin x \quad \Rightarrow \quad \frac{dy}{dx} = k\cos x$$
$$y = k\cos x \quad \Rightarrow \quad \frac{dy}{dx} = -k\sin x$$
$$y = k\tan x \quad \Rightarrow \quad \frac{dy}{dx} = k\sec^2 x$$