Describing Motion in 2-D

Starter

1. **(Review of last lesson)** In a chemical reaction, a chemical X has a concentration of x moles per litre. x is known to satisfy the differential equation $\frac{dx}{dt} = k(a-x)^2$, where a and k are constants. Given that the initial concentration is zero, find a formula for x at time t, expressing your answer in terms of a and k.

Notes

At AS level we studied motion in 1-dimension but at A2 level we look at motion in 2-dimensions. Therefore, we need to use vectors in order to describe the motion.
The same equations we used in 1-dimension are the same in 2-dimensions

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{time}}$$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{time}}$$

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

N.B. Distance is a scalar (i.e. a number) and displacement is a vector.
The path an object follows is called its trajectory.
The velocity vector gives the direction of motion of the object.

Useful vector information from AS

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ "the vector from A to B is $\mathbf{b} - \mathbf{a}$"

Distance between two points A and B is $\left|\overrightarrow{AB}\right| = \left|\mathbf{b} - \mathbf{a}\right|$.

The ***direction*** of a vector is the angle it makes with the $x-$axis, measured in an anticlockwise direction.

E.g. 1 A particle travels from the point $A\begin{pmatrix}-5\\-3\end{pmatrix}$ to $B\begin{pmatrix}7\\2\end{pmatrix}$ in 2 seconds.

It then travels from $B\begin{pmatrix}7\\2\end{pmatrix}$ to $C\begin{pmatrix}4\\6\end{pmatrix}$ in 4 seconds. Calculate:

(a) the displacement and the average velocity from A to B
(b) the distance travelled and the average speed from B to C
(c) the average velocity from A to C
(d) the average speed from A to C.

Working: (a) The vector from A to B is $\mathbf{b} - \mathbf{a}$

So displacement from A to B is $\begin{pmatrix}7\\2\end{pmatrix} - \begin{pmatrix}-5\\-3\end{pmatrix} = \begin{pmatrix}12\\5\end{pmatrix}$

$$\text{Average velocity} = \frac{\begin{pmatrix}12\\5\end{pmatrix}}{2} = \begin{pmatrix}6\\2.5\end{pmatrix}$$

Further examples
Read worked example 19.2 on p427
Read worked example 19.4 on p428

Solutions to Starter and E.g.s

Exercise
p429 19A Qu 1i, 2i, 3-8

Summary

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{time}}$$

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$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$