Differentiating Parametric Equations

Starter

1. (Review of last lesson)
   Transform the parametric curve $x = \tan\theta$, $y = \sec\theta$ into cartesian form.

2. Given that $x = 5t$, $y = t^2$, express $\dfrac{dy}{dx}$ in terms of t.
   Hint: remember the chain rule $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

Notes

Parametric differentiation simply uses the chain rule.

Chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$ ***or*** $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

E.g. 1 Find $\dfrac{dy}{dx}$ in terms of t for the following curves:

(a) $x = e^{3t}$, $y = 4t^3 - 2t^2$

(b) $x = \cos t - \sin 5t$, $y = 4\sin t$

Working: (a) $\dfrac{dx}{dt} = e^{3t}$ and $\dfrac{dy}{dt} = 12t^2 - 4t = 4t(3t - 1)$

Using $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$: $\dfrac{dy}{dx} = \dfrac{4t(3t-1)}{3e^{3t}}$

E.g. 2 The curve C is defined by the parametric equations $x = t^2 - 1$ and $y = t^3 - 3t + 4$.

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find the coordinates of the turning points.

E.g. 3 Find the equation of the normal of the curve $x = \sin 2t$, $y = t\cos t + 2\sin t$ at $t = \pi$.

Video: Differentiating parametric functions
Video: Parametric functions (tangents and normals)

Solutions to Starter and E.g.s

Exercise

p262 12C Qu 1i, 2i, 3-8

Summary

Chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$ or $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$