

Differentiation of a^x

Starter

1. **(Review of last lesson)**
Find the gradient of the curve $e^x + 2 \ln y = y^3$ at the point where $y = 1$.
2. **(Review of last lesson)** If $\tan y = 2x$, find $\frac{dy}{dx}$ in terms of x .

Notes

Using implicit differentiation, we can now differentiate functions involving a^x .

E.g. 1 By firstly taking natural logarithms of both sides and then using one of the laws of logarithms, differentiate $y = 3^x$.

Working:

$$\begin{aligned} \ln y &= \ln 3^x && \text{taking ln of both sides} \\ \ln y &= x \ln 3 && \text{using the 3rd law of logs} \\ \text{N.B. } \ln 3 &\text{ is now the coefficient of } x && \\ \frac{1}{y} \frac{dy}{dx} &= \ln 3 && \text{differentiating with respect to } x \\ \frac{dy}{dx} &= y \ln 3 && \text{rearranging} \\ \frac{dy}{dx} &= 3^x \ln 3 && \text{replace } y \text{ by } 3^x \end{aligned}$$

E.g. 2 Differentiate the following functions with respect to x :

(a) $y = 7^x$ (b) $y = 5 \times 8^x$ (c) $y = 2^{x^3}$ (d) $y = x^x$

In general: if $y = a^{f(x)}$ then $\frac{dy}{dx} = f'(x)a^{f(x)} \ln a$

[Video \(differentiating \$a^x\$ \):](#)
[Video \(differentiating \$x^x\$ \):](#)

[Derivatives of \$a^x\$](#)
[Derivatives of \$x^x\$](#)

[Differentiation EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p213 10D Qu 4i, 11

Summary

$$y = a^{f(x)} \Rightarrow \frac{dy}{dx} = f'(x)a^{f(x)} \ln a$$