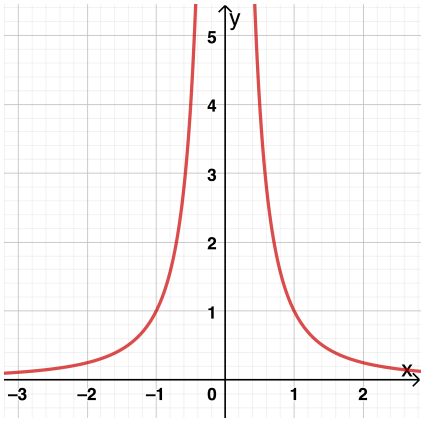
Domain and range

Starter

1. Decide whether the mapping $y = \frac{1}{x^2}$ is a function or just a mapping. If it is a function, decide whether it is a one-to-one function and whether it has an inverse.



Notes

The set of ***input values*** (i.e. the $x-$values) is called the ***domain*** of a function.

The set of ***output values*** (i.e. the $y-$values) is called the ***range*** of a function.

When the domain can take any real numbers, we write $x \in \mathbb{R}$ i.e. "x is a member of the real numbers". Domains and ranges can either be written as what the values can take (e.g. $x \geq 0$) or what the values cannot take (e.g. $y \neq 5$).

The range of a function can often be found by sketching the graph.

E.g. 1 State the domain and range for each of these functions:
(a) $y = 2x - 7$ (b) $y = e^x$ (c) $y = \tan x$
Hint: sketch the graph if you are having difficulty.

Working: (a) Domain: $x \in \mathbb{R}$
Range: $y \in \mathbb{R}$

E.g. 2 (a) By completing the square, write $f(x) = x^2 + 6x + 4$ in the form $(x + a)^2 + b$.
(b) Sketch the graph of $y = f(x)$ and state its range for $x \in \mathbb{R}$.

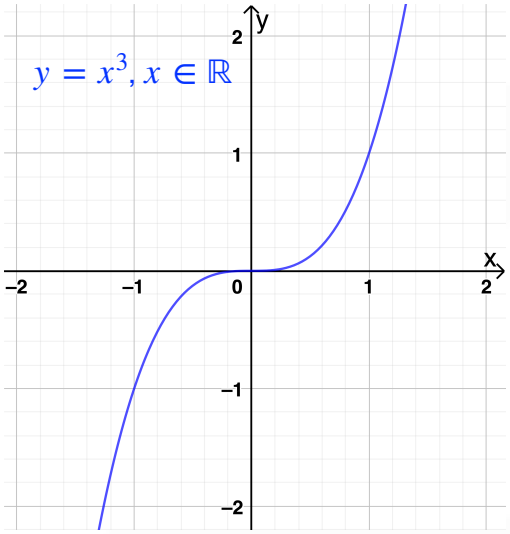
N.B. The range of quadratic can be found by first finding the coordinates of its vertex.

Domain restriction

Domain restriction is when the set of ***input values*** is ***reduced to a specific set of values***.

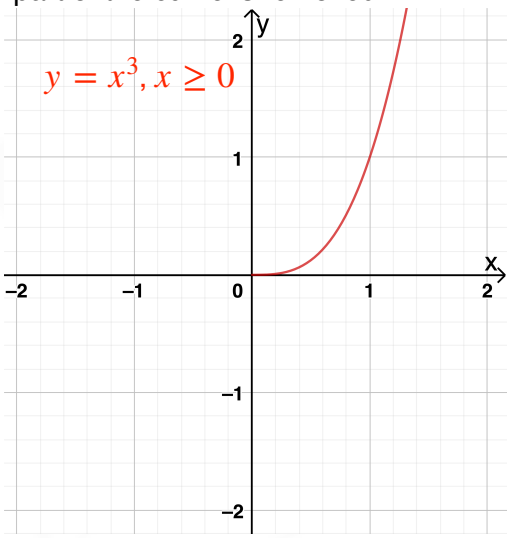
No domain restriction

The graph of $y = x^3$ has an unrestricted domain i.e. x can take any real values



Domain restriction

The domain has been restricted to $x \geq 0$ so part of the curve is removed



E.g. 3 State the range for the function $y = \dfrac{1}{x}$ when the domain is:

(a) $x \in \mathbb{R}$ (b) $x > 2$

Hint: sketch the graph to help you.

E.g. 4 Find the largest possible domain and the corresponding range of these functions:

(a) $y = \sqrt{x^2 - x - 2}$ (b) $y = 6 + \sqrt{28 + 3x - x^2}$

Write your answers in set notation.

Working: (a) To be able to square root a function, it must be greater than or equal to zero.

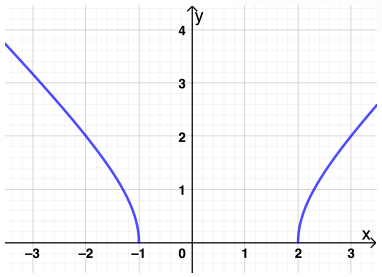
i.e. $x^2 - x - 2 \geq 0 \quad \Rightarrow \quad (x + 1)(x - 2) \geq 0$

Critical values are $x = -1$ and $x = 2$.

$\geq 0 \quad \Rightarrow$ above the $x-$axis so $x \leq -1$ and $x \geq 2$.

Hence, the domain is $\{x : x \leq -1\} \cup \{x : x \geq 2\}$.

As $x \to \infty$, $x^2 - x - 2 \to \infty$ so the range is $\{y : y \geq 0\}$.



Video: Domain and range

Domain and range EQ

Solutions to Starter and E.g.s

Exercise
p20 2B Qu 1i, 2i, 3i, 4i, 5-9, (10-12 red)

Summary
The set of ***input values*** (i.e. the $x-$values) is called the ***domain*** of a function.
The set of ***output values*** (i.e. the $y-$values) is called the ***range*** of a function.

Domains and ranges can either be written as what the values can take (e.g. $x \geq 0$) or what the values cannot take (e.g. $y \neq 5$).

Domain restriction is when the set of ***input values*** is ***reduced to a specific set of values***. This can be used with many-to-one functions to make them one-to-one functions so that they have an inverse.