Double angle identities

Starter

1. Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find the exact value of:
   (a) $\sin(A - B)$ (b) $\cos(A + B)$

2. Using the compound angle identities and by setting B equal to A, find expressions for:
   (a) $\sin 2A$ (b) $\cos 2A$ (c) $\tan 2A$

Notes

The double angle identities are:

$$\sin 2A \equiv 2\sin A\cos A$$
$$\cos 2A \equiv \cos^2 A - \sin^2 A$$
$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

It is mathematically better to write the identities with an equivalent symbol, $\equiv$, rather than the equals symbol, $=$, because the left-hand side is equal to the right-hand side regardless of the value of θ.

E.g. 1 Starting with $\cos 2A \equiv \cos^2 A - \sin^2 A$ and using the identity involving $\cos A$ and $\sin A$, find an identity for $\cos 2A$ involving:
(a) $\cos A$ (b) $\sin A$

Working: (a) $\cos^2 A + \sin^2 A \equiv 1 \quad \Rightarrow \quad \sin^2 A \equiv 1 - \cos^2 A$

Substituting: $\cos 2A \equiv \cos^2 A - (1 - \cos^2 A)$

$\cos 2A \equiv 2\cos^2 A - 1$

While $\cos 2A \equiv \cos^2 A - \sin^2 A$ is the original identity, questions often require the use of:
either $\cos 2A \equiv 2\cos^2 A - 1$ ***or*** $\cos 2A \equiv 1 - 2\sin^2 A$

E.g. 2 Solve these equations for $0° \le \theta \le 360°$:
(a) $\sin 2\theta = \cos\theta$ (b) $2\cos 2\theta = 5 - 13\sin\theta$
(c) $\tan 2\theta + \tan\theta = 0$ (d) $\sin 2\theta = 2\cos 2\theta$
If not exact, give your angles to 3 s.f..

Working: (a) Using $\sin 2\theta = 2\cos\theta\sin\theta$: $2\cos\theta\sin\theta = \cos\theta$

$2\cos\theta\sin\theta - \cos\theta = 0$

N.B. ***Do not divide by*** $\cos\theta$ otherwise solutions will be lost.

Factorise: $\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0 \quad \Rightarrow \quad \theta = 90°$ or $270°$

$2\sin\theta - 1 = 0 \quad \Rightarrow \quad \sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$ or $150°$

So $\theta = 30°, 90°, 150°$ or $270°$.

When proving identities, it is usual to start with the expression on the left-hand side and to manipulate it over a series of steps until it becomes the expression on the right-hand side.

E.g. 3 Prove the identities:

(a) $$\frac{\cos 2A}{\cos A + \sin A} \equiv \cos A - \sin A$$ (b) $$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

Working: (a)
$$\begin{aligned}\frac{\cos 2A}{\cos A + \sin A} &\equiv \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} \\ &\equiv \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A + \sin A} \\ &\equiv \cos A - \sin A\end{aligned}$$

E.g. 4 Find the values of $\sin 2\theta$ and $\cos 2\theta$ given that $\sin\theta = \frac{3}{5}$ and θ is obtuse.

Video: Double angle identities (proving identities)
Video: Double angle identities (solving equations)

Double angle identities EQ

Solutions to Starter and E.g.s

Exercise
p172 8B Qu 1i, 2, 3, 4ac, 5ac, 6ac, 7-10, (11-15 red)

Summary
The double angle identities are:

$$\begin{aligned}\sin 2A &\equiv 2\sin A\cos A \\ \cos 2A &\equiv \cos^2 A - \sin^2 A \\ &\equiv 2\cos^2 A - 1 \\ &\equiv 1 - 2\sin^2 A \\ \tan 2A &\equiv \frac{2\tan A}{1 - \tan^2 A}\end{aligned}$$