

Double angle identities

Starter

1. Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find the exact value of:
- (a) $\sin(A - B)$ (b) $\cos(A + B)$
2. Using the compound angle identities and by setting B equal to A , find expressions for:
- (a) $\sin 2A$ (b) $\cos 2A$ (c) $\tan 2A$

Notes

The double angle identities are:

$$\begin{aligned}\sin 2A &\equiv 2 \sin A \cos A \\ \cos 2A &\equiv \cos^2 A - \sin^2 A \\ \tan 2A &\equiv \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

It is mathematically better to write the identities with an equivalent symbol, \equiv , rather than the equals symbol, $=$, because the left-hand side is equal to the right-hand side regardless of the value of θ .

E.g. 1 Starting with $\cos 2A \equiv \cos^2 A - \sin^2 A$ and using the identity involving $\cos A$ and $\sin A$, find an identity for $\cos 2A$ involving:

- (a) $\cos A$ only (b) $\sin A$ only

Working:

$$\begin{aligned}\text{(a) } \cos^2 A + \sin^2 A &\equiv 1 &\Rightarrow & \sin^2 A \equiv 1 - \cos^2 A \\ \text{Substituting:} & & \cos 2A &\equiv \cos^2 A - (1 - \cos^2 A) \\ & & \cos 2A &\equiv 2 \cos^2 A - 1\end{aligned}$$

While $\cos 2A \equiv \cos^2 A - \sin^2 A$ is the original identity, questions often require the use of:
either $\cos 2A \equiv 2 \cos^2 A - 1$ **or** $\cos 2A \equiv 1 - 2 \sin^2 A$

E.g. 2 Solve these equations for $0^\circ \leq \theta \leq 360^\circ$:

- (a) $\sin 2\theta = \cos \theta$ (b) $2 \cos 2\theta = 5 - 13 \sin \theta$
(c) $\tan 2\theta + \tan \theta = 0$ (d) $\sin 2\theta = 2 \cos 2\theta$

If not exact, give your angles to 3 s.f..

Working:

$$\begin{aligned}\text{(a) Using } \sin 2\theta &= 2 \cos \theta \sin \theta: & 2 \cos \theta \sin \theta &= \cos \theta \\ & & 2 \cos \theta \sin \theta - \cos \theta &= 0\end{aligned}$$

N.B. Do not divide by $\cos \theta$ otherwise solutions will be lost.

Factorise: $\cos \theta(2 \sin \theta - 1) = 0$
 $\cos \theta = 0 \Rightarrow \theta = 90^\circ \text{ or } 270^\circ$

$$2 \sin \theta - 1 = 0 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ \text{ or } 150^\circ$$

So $\theta = 30^\circ, 90^\circ, 150^\circ \text{ or } 270^\circ$.

When proving identities, it is usual to start with the expression on the left-hand side and to manipulate it over a series of steps until it becomes the expression on the right-hand side.

E.g. 3 Prove the identities:

$$(a) \quad \frac{\cos 2A}{\cos A + \sin A} \equiv \cos A - \sin A \qquad (b) \quad \cos 3A \equiv 4 \cos^3 A - 3 \cos A$$

Working:

$$(a) \quad \frac{\cos 2A}{\cos A + \sin A} \equiv \frac{\cos^2 A - \sin^2 A}{(\cos A + \sin A)(\cos A - \sin A)} \\ \equiv \frac{\cos A + \sin A}{\cos A - \sin A} \\ \equiv \cos A - \sin A$$

E.g. 4 Find the values of $\sin 2\theta$ and $\cos 2\theta$ given that $\sin \theta = \frac{3}{5}$ and θ is obtuse.

Video: [Double angle identities \(proving identities\)](#)

Video: [Double angle identities \(solving equations\)](#)

[Double angle identities EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p172 8B Qu 1i, 2, 3, 4ac, 5ac, 6ac, 7-10, (11-15 red)

Summary

The double angle identities are:

$$\begin{aligned} \sin 2A &\equiv 2 \sin A \cos A \\ \cos 2A &\equiv \cos^2 A - \sin^2 A \\ &\equiv 2 \cos^2 A - 1 \\ &\equiv 1 - 2 \sin^2 A \\ \tan 2A &\equiv \frac{2 \tan A}{1 - \tan^2 A} \end{aligned}$$