Equation of the Trajectory

Starter

1. (Review of last lesson) At the point of being hit, a golf ball has an initial speed of 22 m/s. It lands 4 s later. Find its angle of projection.
Hint: how long does the ball take to reach its maximum height?

N.B. Using SUVAT equations (or by integrating) we can find the following:

	Horizontally	Vertically
Acceleration	$a_x = 0$	$a_y = -g$
Velocity	$v_x = u\cos\theta$	$v_y = u\sin\theta - gt$
Displacement	$s_x = ut\cos\theta$	$s_y = ut\sin\theta - \frac{1}{2}gt^2$

2. Show that a particle projected from the ground with velocity u at an angle of θ to the horizontal:
   (a) reaches its maximum height when $t = \dfrac{u\sin\theta}{g}$.
   (b) reaches a maximum height of $\dfrac{u^2\sin^2\theta}{2g}$
   (c) hits the ground when $t = \dfrac{2u\sin\theta}{g}$.
   (d) has a range of $\dfrac{u^2\sin 2\theta}{g}$
   (e) Given that the range, R, is given by $R = \dfrac{u^2\sin 2\theta}{g}$, prove by differentiation that the greatest range is when $\theta = 45^{\circ}$. Remember u and g are constants.

Notes

Trajectory of a projectile

The coordinates of a projectile are given by:

$$x = ut\cos\theta \qquad\qquad y = ut\sin\theta - \frac{1}{2}gt^2$$

Assuming that u and θ are fixed once the projectile is in motion, these equations can be considered parametric equations. Therefore, to find the equation of the trajectory we need to eliminate t from them.

Rearrange the equation for x to get $\qquad t = \dfrac{x}{u\cos\theta}$

Replace t by $\dfrac{x}{u\cos\theta}$ in the equation for y to get:

$$y = u\frac{x}{u\cos\theta}\sin\theta - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad \text{equation of trajectory}$$

N.B. This is ***not*** on the formula page.
As can be seen, the equation is a quadratic.

E.g. 1 A projectile is fired from the origin with speed 3 m/s, at an angle α above the horizontal. The projectile passes through the point $(3,\ 1)$ m. Show that $1 = 3\tan\alpha - \dfrac{g}{2\cos^2\alpha}$.

Hint: use the equation for the trajectory.

E.g. 2 A golf ball is struck with an angle of 60^{o} and passes through the point $(4,\ 2)$. What was the initial speed of the ball?

Limitations of the formula for the trajectory
The path of a projectile is not a perfect parabola since air resistance is not taken into account.
Diagram on p458

Video: Equation of trajectory
Video: Finding speed and direction

Projectiles EQ

Solutions to Starter and E.g.s

Exercise
p460 20B Qu 1-3, 4-6 (red)

Summary

The equation of the trajectory of a projectile is given by $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$