

## Coefficient of Friction

### Notes

- Smooth  $\Rightarrow$  no frictional force  
Rough  $\Rightarrow$  frictional force present

### Key ideas about friction

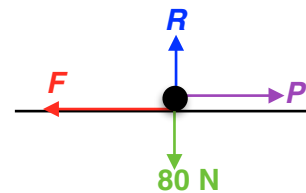
1. Friction acts in the **opposite direction to motion**
2. The maximum value of the frictional force is called the **limiting friction** value or  $F_{lim}$ .
3. By N3L, **as the force pushing** an object **increases** so does the frictional force resisting motion. The **friction force** opposing motion **increases** until it reaches  $F_{lim}$ , i.e. the maximum frictional force.
4. If an object is **on the point of moving**, the object is said to be in **limiting equilibrium**.
5. The limiting frictional force between two surfaces is proportional to the normal contact force. If the limiting frictional is  $F_{lim}$  and the normal contact force is  $R$ , then  $F_{lim} = \mu R$ , where  $\mu$  is a constant. The constant  $\mu$  is called the **coefficient of friction** and has no units.
6. **Moving** objects: the frictional force is  $F_{lim} = \mu R$ .
7. **Stationary** objects: the frictional force is  $F_{lim} \leq \mu R$ .
8. For all rough surfaces:  $0 < \text{Frictional force} \leq \mu R$ .
9. If contact between two surfaces is smooth,  $\mu = 0$ .

**N.B.** Experiments indicate that friction when in motion, friction is slightly less than the limiting friction value.

**E.g. 1** A horizontal force  $P$  newtons is applied to a body of weight 80 N, standing in rough contact with a horizontal plane. The coefficient of friction between the body and the plane is 0.5. What is the magnitude of the frictional force,  $F$ , when:

- (a)  $P = 10$  N
- (b)  $P = 40$  N
- (c)  $P = 50$  N?

State in each case whether or not the body moves.



### Solving friction problems

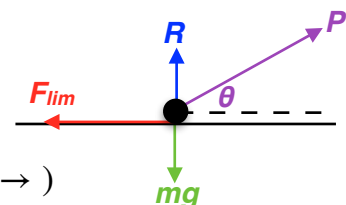
When doing friction questions you can do the following three things:

- $R(\uparrow)$  — resolve vertically to find  $R$ , the normal contact force.
- $F_{lim} = \mu R$  — to find the limiting (maximum) frictional force.
- $R(\rightarrow)$  or  $F = ma(\rightarrow)$  — resolve or use  $F = ma$  horizontally

**N.B.** The difference is just which order you do them in — often this does not matter.

**E.g. 2** A particle of mass  $m$  kg lies on a rough horizontal plane. The coefficient of friction between the particle and the plane is  $\mu$ . A force of  $P$  N acts at an angle  $\theta$  above the horizontal. For the situations where the particle is:

- (a) accelerating and
- (b) in limiting equilibrium or moving with constant speed:
  - (i) Resolve vertically,  $R(\uparrow)$
  - (ii) Use  $F_{lim} = \mu R$
  - (iii) Use  $F = ma(\rightarrow)$  or resolve horizontally  $R(\rightarrow)$



**E.g. 3** A small block of weight 32 N is lying in rough contact on a horizontal plane. A horizontal force of  $P$  newtons is applied to the block until it is just about to move the block.

- (a) If  $P = 8$ , find the coefficient of friction,  $\mu$ , between the block and the plane.
- (b) If  $\mu = 0.4$ , find the value of  $P$ .

**E.g. 4** A particle of mass 5 kg, resting on a rough plane, is acted on by a force of 23 N. The coefficient of friction between the particle and surface is  $\frac{1}{3}$ .

- Find the acceleration when the force acts horizontally.
- If the force acts at  $20^\circ$  to the horizontal will the acceleration increase or decrease?
- Find the new acceleration.

**Finding the contact force between two surfaces**

The contact force,  $C$ , is the resultant from the frictional and the normal contact force.

$$C = \sqrt{R^2 + F_{lim}^2}$$

**E.g. 5** A particle of mass 12 kg is being pulled along a rough surface by a horizontal force of  $P$  N such that its acceleration is  $3 \text{ m/s}^2$ . The coefficient of friction between the particle and surface is  $\frac{3}{4}$ .

- Find the value of  $P$ .
- Find the magnitude and direction of the contact force.

**Exercise**

- A person tries to pull a small cupboard across a floor. The mass of the cupboard is 80 kg and the coefficient of friction is 0.5. Describe what happens if the cupboard is pulled with a horizontal force of:
  - 200 N,
  - 400 N.
- A small block of weight 24 N rests in rough contact with a horizontal plane. A light string is attached to the block and is inclined at  $30^\circ$  to the plane. The block is just about to slip when the tension in the string is 12 N. Find:
  - the coefficient of friction between the block and the plane
  - the magnitude of the contact force.
- A block of weight 20 N is at rest on a horizontal surface. When a force of magnitude 12 N is applied to the block at an angle of  $30^\circ$  above the horizontal via a string, it is on the point of moving. Find:
  - the coefficient of friction between the block and the surface
  - the magnitude of the contact force.
- A box of mass 2 kg is placed on a table. A string attached to the box passes over a smooth peg at the edge of the table, and a ball of mass 1 kg is tied to the other end.
  - If the coefficient of friction between box and table is 0.2, find the acceleration of the box and the ball.
  - Find the magnitude of the contact force between the box and the table.
- A block of mass 5 kg is being pulled along a rough surface by a force,  $D$ , which acts at  $30^\circ$  to the horizontal. The frictional force resisting the motion is 15 N. Find:
  - the force  $D$  given that the block is accelerating at  $3 \text{ m/s}^2$  and
  - the normal contact force between the block at the rough surface.
  - How large does  $D$  have to be for the block to leave the surface?
- A truck is pulled along a horizontal track by two cables against resistance forces totally 1100 N, with an acceleration of  $0.8 \text{ m/s}^2$ . One cable is horizontal and has a tension of 800 N. The other cable is inclined at  $40^\circ$  to the track and has a tension of 1000 N. Find:
  - the mass of the truck
  - the vertical force exerted by the track on the truck.
- A small block of mass 5 kg is being pulled along a rough horizontal plane by a string inclined at  $60^\circ$  to the plane. There is a frictional force of 18 N and the acceleration of the block is  $3 \text{ m/s}^2$ .
  - Draw a diagram, marking on all the forces that act on the block.
 Find
  - the tension in the string and
  - the normal reaction exerted by the plane on the block.