Geometric Sequences

Starter

1. **(Review of last lesson)** Sticky tape is wound into a spiral on a reel. The first complete loop is 25.00 cm, the second 25.05, the next 25.10 and so on, forming 100 loops. What is the total length of the tape on the reel?

2. **(Review of last lesson)**

Find the value of N which makes the sum $\sum_{r=1}^{N}(2+5r)$ greater than 500.

Notes

A geometric progression (GP) is when successive terms are found by multiplying the previous term by the same number. This number could be an integer, fraction, decimal, positive or negative and is called the common ratio.

E.g. 2, 6, 18, … common ratio $= \frac{6}{2} = \frac{18}{6} = 3$

5, −10, 20, −40, … common ratio is -2

64, 32, 16, 8,… common ratio $= \frac{32}{64} = \frac{1}{2}$

E.g. 1 (a) Given that the first term of a general GP is a and the common ratio is r, write down the first 4 terms.

(b) Hence express the the $n-$th term, u_n, in terms of a, n and r.

Working: (a) $a, ar, ar^2, ar^3, \ldots$

(b) $u_n = ar^{n-1}$

The nth term of a geometric progression is given by: $u_n = ar^{n-1}$

N.B. $r = \frac{u_2}{u_1}$ $r = \frac{u_3}{u_2}$ $r = \frac{u_4}{u_3}$ $r = \frac{u_{n+1}}{u_n}$

E.g. 2 Find an expression for the nth term of the sequence 2, 6, 18, 54.

E.g. 3 Find the number of terms in these geometric sequences:

(a) 4, 8, 16, …, 262144 (b) 3, −6, 12, …, −98304

N.B. Ignore the −ve sign when finding the number of terms (cannot find the log of a −ve number).

E.g. 4 Find the common ratio and the first term of a geometric sequence whose third term is 6 and whose 7th term is 486.

Video: Geometric progressions

Solutions to Starter and E.g.s

Exercise
p78 4E Qu 1i, 2i, 3-11

Summary
Geometric progression (GP): $a, ar, ar^2, ar^3, \ldots$
$n-$term of a GP: $u_n = ar^{n-1}$
a = first term r = common ratio
N.B. $r = \frac{u_2}{u_1}$ $r = \frac{u_3}{u_2}$ $r = \frac{u_4}{u_3}$ $r = \frac{u_{n+1}}{u_n}$