

Geometric Series

Starter

1. **(Review of last lesson)** Winston invests a sum of money at 6% per annum. How many complete years does it take him to double his money?
2. **(Review of last lesson)** A geometric sequence has first term 25000 and common ratio 0.8. Which term is the first to be below 1000?

Notes

What is the sum to n terms of a geometric series?

$$S_n = a + ar + ar^2 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1} \quad - (1)$$

Multiplying the equation by r gives:

$$rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n \quad - (2)$$

Equation (2) — Equation (1):

$$\begin{aligned} rS_n - S_n &= ar^n - a \\ S_n(r - 1) &= a(r^n - 1) \\ S_n &= \frac{a(r^n - 1)}{r - 1} \quad - \text{ use when } |r| > 1 \end{aligned}$$

Alternatively:

Equation (1) — Equation (2):

$$\begin{aligned} S_n - rS_n &= a - ar^n \\ S_n(1 - r) &= a(1 - r^n) \\ S_n &= \frac{a(1 - r^n)}{1 - r} \quad - \text{ use when } |r| > 1 \end{aligned}$$

E.g. 1 Find the sum of the first 9 terms of the series 2, 10, 50.

E.g. 2 The sum of the first n terms of a geometric series is 196 605. Given that the common ratio is 2 and the first term is 3. Find n .

E.g. 3 The 3rd term of a geometric series is 6 and the 8th term is 192. Find the sum of the first 15 terms.

Hint: find a and r first.

E.g. 4 Find $\sum_{n=1}^{12} (3 \times 4^n)$

Video: [Sum of geometric series](#)

[Solutions to Starter and E.g.s](#)

Exercise

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Summary

Geometric progression (GP): a, ar, ar^2, ar^3, \dots

n -term of a GP: $u_n = ar^{n-1}$

a = first term r = common ratio

N.B. $r = \frac{u_2}{u_1}$ $r = \frac{u_3}{u_2}$ $r = \frac{u_4}{u_3}$ $r = \frac{u_{n+1}}{u_n}$

Sum to n terms: $S_n = \frac{a(r^n - 1)}{r - 1}$ — use when $|r| > 1$

$S_n = \frac{a(1 - r^n)}{1 - r}$ — use when $|r| < 1$