

Indefinite Integration by Substitution

Starter

1. **(Review of last lesson)** Find: (a) $\int \cos(2x + 7)dx$ (b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sec^2 x dx$

2. (a) Use the method "Let $u = \dots$ " to find $\int (3x - 2)^7 dx$.

(b) Use a similar method to (a), find:

(i) $\int x(x^2 - 5)^7 dx$ (ii) $\int x(x - 5)^7 dx$

N.B. It is usually better to start the factorisation process when the function is in terms of u .

Notes

When there is no quick route to integrate a function, integration by substitution can be used.

The first line is always "Let $u = \dots$ " and there is no quick and easy method to replace it.

N.B. Substitutions may or may not be given.
Usually $u =$ the function in the bracket.

Sometimes functions will cancel top and bottom, other times not.

No cancelling

E.g. 1 Find $\int (x + 1)(x + 3)^5 dx$.

Working: Let $u = x + 3 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

$$\begin{aligned} \int (x + 1)(x + 3)^5 dx &= \int (x + 1)u^5 du && \text{replace } x + 3 \text{ and } dx \\ &= \int (u - 3 + 1)u^5 du && \text{since } x = u - 3 \\ &= \int (u - 2)u^5 du \\ &= \int (u^6 - 2u^5) du && \text{expand the brackets} \\ &= \frac{1}{7}u^7 - \frac{1}{3}u^6 + c && \text{integrate with respect to } u \\ &= \frac{1}{21}u^6(3u - 7) + c \\ &= \frac{1}{21}(x + 3)^6(3(x + 3) - 7) + c && \text{replace } u \\ &= \frac{1}{21}(3x + 2)(x + 3)^6 + c \end{aligned}$$

Cancelling

E.g. 2 Use a suitable substitution to find $\int x\sqrt{x^2 + 1} dx$.

Working: Let $u = x^2 + 1$ so $\frac{du}{dx} = 2x$ and $dx = \frac{du}{2x}$

$$\begin{aligned}\int x\sqrt{x^2 + 1} dx &= \int x u^{\frac{1}{2}} \frac{du}{2x} && \text{replace } x^2 + 1 \text{ by } u \text{ and } dx \text{ by } \frac{du}{2x} \\ &= \frac{1}{2} \int u^{\frac{1}{2}} du && \text{the } x \text{ cancels top and bottom} \\ &= \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + c && \text{integrate} \\ &= \frac{1}{3} \sqrt{(x^2 + 1)^3} + c && \text{replace } u \text{ by } x^2 + 1\end{aligned}$$

Or:

Let $u^2 = x^2 + 1$ so $2u \frac{du}{dx} = 2x$ and $dx = \frac{u}{x} du$

$$\begin{aligned}\int x\sqrt{x^2 + 1} dx &= \int x u \frac{u}{x} du \\ &= \int u^2 du \\ &= \frac{1}{3} u^3 + c \\ &= \frac{1}{3} \sqrt{(x^2 + 1)^3} + c\end{aligned}$$

E.g. 3 Use a suitable substitution to integrate:

(a) $\int x(2x^2 - 1)^4 dx$

(b) $\int \frac{\sin x}{\cos^3 x} dx$

[Video: Integration by substitution](#)
[Video: Integration by substitution involving square roots](#)

[Integration by substitution EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

p229 11C Qu 1i, 2i, 3i

Summary

Let $u = \dots$

Make sure all terms in x and dx are replaced by terms in u and du before integrating.