

Infinite Geometric Series

Starter

- (Review of last lesson)** Colin leaves school and starts a job with a salary of £16,000 in the first year. He receives an annual increase of 4% per annum.
 - Assuming that he works for 40 years, calculate:
 - the amount he will earn in his 40th year
 - the total amount that he will earn over the 40 year period.
 - How many full years will he need to work for his total earnings to exceed more than £1 million?
- (Review of last lesson)**
Find the value of N which makes the sum $\sum_{r=1}^N (2 \times 5^r)$ (GP) greater than 50000.
- True or false: the sum of an infinite number of positive numbers is always infinite.
- What is the sum to an infinite number of terms of an arithmetic series?

Notes

The sum to infinity of a geometric series is **finite** (i.e. interesting) when the **terms converge** (i.e. their **magnitude gets smaller**).

This happens when the **common ratio is between -1 and 1** :

$$-1 < r < 1 \quad \text{or} \quad |r| < 1$$

E.g. 1 Decide whether the geometric series $\frac{2}{3} + \frac{4}{9} + \frac{8}{27}$ converges.

Deriving the formula for the sum to infinity of a GP

$$\text{From } S_n = \frac{a(1 - r^n)}{1 - r}$$

When $-1 < r < 1$, as $n \rightarrow \infty$, $r^n \rightarrow 0$:

$$\lim_{n \rightarrow \infty} S_n = \frac{a(1 - 0)}{1 - r} = \frac{a}{1 - r}$$

In practice we write, $S_\infty = \frac{a}{1 - r}$ this is the formula for the **sum to infinity** of a **converging geometric series**

E.g. 2 Find the sum to infinity of $243 + 81 + 27 + \dots$

E.g. 3 The sum to infinity of a geometric series is 45 and the common ratio is $\frac{3}{5}$. Find the first term.

E.g. 4 The sum to infinity of a geometric series is 6. If the first term of the series is equal to the common ratio, find the value of the tenth term. Express your answer in index form.

E.g. 5 A ball is dropped from a height of 1 m onto a table top. It always rises to a height of 0.9 of the height from which it was dropped. How far does it travel in total until it stops bouncing?
Hint: consider downwards and upwards motion separately.

E.g. 6* The 5th, 9th and 12 terms of an arithmetic series, with common difference of 3 are the 1st 3 terms of a geometric series. Show that the geometric series is convergent and find its sum to infinity.

Video: [Sum to infinity](#)

[Solutions to Starter and E.g.s](#)

Exercise

p84 4G Qu 1i, 2i, 3-11

Summary

Geometric progression (GP): a, ar, ar^2, ar^3, \dots

n -term of a GP: $u_n = ar^{n-1}$

a = first term r = common ratio

N.B. $r = \frac{u_2}{u_1}$ $r = \frac{u_3}{u_2}$ $r = \frac{u_4}{u_3}$ $r = \frac{u_{n+1}}{u_n}$

Sum to n terms: $S_n = \frac{a(r^n - 1)}{r - 1}$ — use when $|r| > 1$

$S_n = \frac{a(1 - r^n)}{1 - r}$ — use when $|r| < 1$

Sum to infinity of converging geometric series: $S_\infty = \frac{a}{1 - r}$

A GP converges when $-1 < r < 1$