Integrals involving Brackets

Starter

1. (a) Differentiate $y = (3x + 5)^8$.

   (b) Using your answer to (a) integrate the function $\int (3x + 5)^7 dx$.

Notes

When integrating functions raised to a power e.g. $(4x + 3)^9$ we can do something similar to the chain rule.

E.g. Find $\int (4x + 3)^9 dx$.

Working: Let $u = 4x + 3 \quad \Rightarrow \quad \frac{du}{dx} = 4 \quad \Rightarrow \quad dx = \frac{du}{4}$

$$\int (4x + 3)^9 dx = \int \frac{1}{4} u^9 du \qquad \textit{replace } 4x + 3 \textit{ and } dx$$

$$= \frac{1}{40} u^{10} + c \qquad \textit{integrate with respect to } u$$

$$= \frac{1}{40} (4x + 3)^{10} + c \qquad \textit{replace } u \textit{ by } 4x + 3$$

N.B. It is not enough to write $\int u^9 dx$ because a function of u cannot be integrated with respect to x

E.g. 1 Find: (a) $\int (3x + 5)^7 dx$ (b) $\int k(ax + b)^n dx$

In general: $\int k(ax + b)^n dx = \frac{k}{a(n + 1)} (ax + b)^{n+1} + c$ where $n \neq 1$

N.B. Always differentiate back mentally to check your answer

E.g. 2 Does $\int (x^2 + 3)^6 dx = \frac{1}{14x} \times (x^2 + 3)^7 + c$?

N.B. The function in the brackets ***must be linear***.
These integrations can be done by inspection i.e. there is no need to show all the "Let $u = \ldots$ working".

E.g. 3 Find: (a) $\int 3(1 - 5x)^6 dx$ (b) $\int \frac{1}{(2x - 9)^6} dx$ (c) $\int \sqrt{7x + 5} dx$

E.g. 4 Find: (a) $\int_1^{1.5} (3-2x)^3 dx$ (b) $\int_0^1 \frac{1}{(6x+1)^3} dx$

E.g. 5 The curve $y = f(x)$ goes through the point $\left(1, \frac{3}{35}\right)$ and $f'(x) = (8-7x)^4$. Find $f(x)$.

Video: Integration by inspection (brackets)

Integration by inspection (brackets) EQ

Solutions to Starter and E.g.s

Exercise
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Summary

$$\int k(ax+b)^n dx = \frac{k}{a(n+1)}(ax+b)^{n+1} + c$$