Integrals involving Trigonometry

Notes

1. **(Review of last lesson)** Find: (a) $\int 8e^{2-7x}dx$ (b) $\int \frac{1}{7x+2}dx$

2. By using "Let $u = \ldots$" find the following:

(a) $\int \sin(2x+1)dx$ (b) $\int \sec^2(6x-5)dx$

(c) $\int \cos(ax+b)dx$ (d) $\int \sin(x^2-7)dx$

Working: (a) Let $u = 2x+1 \quad \Rightarrow \quad \frac{du}{dx} = 2 \quad \Rightarrow \quad \frac{du}{2} = dx$

$$\int \sin(2x+1)dx = \int \frac{1}{2}\sin u\,du \qquad \textit{replace } 2x+1 \textit{ and } dx$$

$$= -\frac{1}{2}\cos u + c \qquad \textit{integrate with respect to } u$$

$$= -\frac{1}{2}\cos(2x+1) + c \qquad \textit{replace } u \textit{ by } 2x+1$$

Notes

In general:

$$\int k\sin(ax+b)dx = -\frac{k}{a}\cos(ax+b) + c$$

$$\int k\cos(ax+b)dx = \frac{k}{a}\sin(ax+b) + c$$

$$\int k\sec^2(ax+b)dx = \frac{k}{a}\tan(ax+b) + c$$

N.B. Always differentiate back to check your answer
The function in the bracket must be linear.
Again, full working with "Let $u = \ldots$" does not need to be shown.

E.g. 1 Find: (a) $\int \sin(9x-4)dx$ (b) $\int \sec^2(3x-11)dx$

E.g. 2 Find: (a) $\int_1^{1.5} \cos(2x-1)dx$ (to 3 s.f.) (b) $\int_0^{\frac{\pi}{6}} (\cos 3x + \sin 2x)dx$

Video: Integrals of sin/cos/tan with brackets

Solutions to Starter and E.g.s

Exercise
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Summary

$$\int k(ax+b)^n dx = \frac{k}{a(n+1)}(ax+b)^{n+1} + c$$

$$\int ke^{ax+b} dx = \frac{k}{a}e^{ax+b} + c$$

$$\int \frac{k}{ax+b} dx = \frac{k}{a}\ln(ax+b) + c$$

$$\int k\sin(ax+b)dx = -\frac{k}{a}\cos(ax+b) + c$$

$$\int k\cos(ax+b)dx = \frac{k}{a}\sin(ax+b) + c$$

$$\int k\sec^2(ax+b)dx = \frac{k}{a}\tan(ax+b) + c$$