Integrals of Trigonometric Functions

Starter

1. **(Review of last lesson)**

A curve has gradient $\frac{4}{3x}$ and passes through the point (2, 7). Susan says the equation of the curve could be either $y = \frac{4}{3}\ln x + 6.08$ or $y = \frac{4}{3}\ln 3x + 4.61$. Tom said she is wrong because the equation must be unique. Who is correct and why?

2. Given that $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\cos x)}{dx} = -\sin x$ and $\frac{d(\tan x)}{dx} = \sec^2 x$, find:

(a) $\int k \sin x\,dx$ (b) $\int k \cos x\,dx$ (c) $\int k \sec^2 x\,dx$.

Notes

Always differentiate back mentally to check your answer is correct…and include the constant of integration.

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

N.B. When using trigonometric in calculus, angles must be in ***radians***.

E.g. 1 Find: (a) $\int \frac{1}{7}\cos x\,dx$ (b) $\int 6\sin x - 7\sec^2 x\,dx$

Working: (a) $\int \frac{1}{7}\cos x\,dx = \frac{1}{7}\sin x + c$

E.g. 2 Find $\int_0^{\frac{\pi}{6}} 5\cos x\,dx$.

E.g. 3 Find k such that $\int_k^{\frac{\pi}{3}} 7\sin x\,dx = \frac{1}{3}$. Give your answer to 3 s.f.

Video: Integrating sin/cos/tan

Solutions to Starter and E.g.s

Exercise

p193 9B Qu 1idef, 3ifg, 5, 6, 9, 12 (not Qu 10)

Qu 12(a) You need to use your calculator to solve $e^x - 5\sin x = 0$.

Video: Solving equations with Classwiz

Make sure your calculator is in radians.

Enter $e^x - 5\sin x = 0$ on your Classwiz using

Alpha >>) to enter "x"

ALPHA >> CALC to enter "="

Then press SHIFT >> CALC to SOLVE

For some reason your calculator throws up an old value for x that is in its memory. Press "=" to get the actual value you want which is 0.263265.

Summary

$$\int Ae^{kx}dx = A\int e^{kx}dx$$

$$\int \frac{P}{Qx}dx = \frac{P}{Q}\int \frac{1}{x}dx = \frac{P}{Q}\ln|x| + c$$

$$\int k\sin x dx = -k\cos x + c$$

$$\int k\cos x dx = k\sin x + c$$

$$\int k\sec^2 x dx = k\tan x + c$$