

Integrals of the form $\frac{f'(x)}{f(x)}$

Starter

1. **(Review of previous A2 material)** Differentiate:
 (a) $y = \ln(3x^2 + 5x)$ (b) $y = \ln(\sin x)$ (c) $y = \ln(5e^x + 2x)$

Hint: For $y = k \ln[f(x)] \Rightarrow \frac{dy}{dx} = \frac{kf'(x)}{f(x)}$

2. Using your answers to question 1, integrate the following:

(a) $\int \frac{10x + 7}{5x^2 + 7x} dx$ (b) $\int \frac{\cos x}{4 + \sin x} dx$ (c) $\int \frac{e^x}{3e^x - 1} dx$

N.B. Remember to use the modulus brackets i.e. $\ln | \quad |$

Notes

When integrating functions that have a numerator and a denominator, see if the **numerator** is a **multiple of the derivative of the denominator**.

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

N.B. Always check your answer by differentiating.

[When in doubt use "Let $u = \dots$ ".]

E.g. 1 Find: (a) $\int \frac{x^3}{1+x^4} dx$ (b) $\int \frac{x-1}{3x^2-6x+1} dx$

Working: (a) $\frac{d}{dx} \ln |1+x^4| = \frac{4x^3}{1+x^4}$ so we need to divide by 4
 $\int \frac{x^3}{1+x^4} dx = \frac{1}{4} \ln |1+x^4| + c$

E.g. 2 Since x^3 is a multiple of the derivative of $1+x^4$, we can find $\int \frac{x^3}{1+x^4} dx$ using

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c.$$

Decide whether these adjustments mean that $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$?

Give your answer as "still works" or "does not work anymore".

(a) $\int \frac{5x^3}{1+x^4} dx$ (b) $\int \frac{x^2}{1+x^4} dx$ (c) $\int \frac{x^3}{1+x^3} dx$
 (d) $\int \frac{x^3}{6+x^4} dx$ (e) $\int \frac{x^3}{1+7x^4} dx$

Working: (a) Still works

E.g. 3 Find: (a) $\int \frac{2x + 3}{x^2 + 3x - 4} dx$ (b) $\int \tan x dx$

Hint: For (b), remember the identity involving sine, cosine and tan.

E.g. 4 Find: (a) $\int \frac{dx}{5x + 2}$ (b) $\int \frac{dx}{1 - 2x}$ (c) $\int \frac{x^3}{3x^4 - 5} dx$

E.g. 5 Find $\int_0^1 \frac{x}{x^2 + 1} dx$, expressing your answer in the form $\ln \sqrt{a}$ where a is an integer.

E.g. 6 Find $\int \frac{1}{x \ln x} dx$.

Hint $\int \frac{1}{x \ln x} dx \equiv \int \frac{\frac{1}{x}}{\ln x} dx$

E.g. 7* Which adjustment(s) need to be made to $\int \frac{4e^x - 15x^3}{4e^{2x} - 5x^6} dx$ so that $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$ can be used?

Video: [Integrals of the form f'\(x\)/f\(x\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

C3/C4 book (purple) p174 Ex 2 Qu 1-12, 13, 14 ace..., 15, 16ac, 17ace

Summary

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$