# Integrating Parametric Equations

**Starter**

1. **(Review of last lesson)** Find $\frac{dy}{dx}$ for the curve $x = \ln 5t,\ y = 3t^2 - t^3$.

2. **(Review of A2 material)** Find $\int_{\frac{\pi}{2}}^{\pi} x \sin 3x\, dx$.

**Notes**

Let $y = f(x)$ be a curve that can be expressed parametrically in the form $x = x(t)$ and $y = y(t)$. Let the corresponding $t-$values of $x_1$ and $x_2$ be $t_1$ and $t_2$ respectively.

We need an expression for $\int_{x_1}^{x_2} y\, dx$ with $dx$ replaced by $dt$.

$$dx = dx \times \frac{dt}{dt} \qquad \textit{since } \frac{dt}{dt} = 1$$
$$= \frac{dx}{dt} dt$$

Then $\int_{x_1}^{x_2} y\, dx = \int_{t_1}^{t_2} y \frac{dx}{dt} dt$

***N.B.*** Remember to change the limits as you change from $dx$ to $dt$
Given the nature of the formula, integration by parts may be required

***E.g. 1*** Find an expression in parametric form that is equivalent to $\int y\, dx$:

(a) $x = \frac{3}{t},\ y = 4t^2$ (b) $x = \sqrt{t},\ y = 3t^2 - 4$

***Working:*** (a) From $x = \frac{3}{t} = 3^{-t}$, $\frac{dx}{dt} = -3t^{-2} = -\frac{3}{t^2}$

$$\int y \frac{dx}{dt} dt = \int 4t^2 \times -\frac{3}{t^2} dt = -\int 12 dt$$

***E.g. 2*** A curve has parametric equations $x = 3t^2,\ y = \frac{5}{t}$, where $t > 0$. Find the value of $\int_3^{75} y\, dx$.

***E.g. 3*** A curve has parametric equations $x = 6t^2,\ y = e^{2t}$, where $t > 0$. Find the value of $\int_0^6 y\, dx$.

**Video A:** Integrating parametric functions
**Video B:** Integrating parametric functions

Solutions to Starter and E.g.s

**Exercise**
p264 12D Qu 1-4

**Summary**

$$\int_{x_1}^{x_2} y\,dx = \int_{t_1}^{t_2} y\frac{dx}{dt}dt$$