Integrating Parametric Equations

Starter

1. **(Review of last lesson)** Find $\frac{dy}{dx}$ for the curve $x = \ln t$, $y = 3t^2 - t^3$.

2. **(Review of A2 material)** Find $\int_{\frac{\pi}{2}}^{\pi} x \sin x dx$.

Notes

Let $y = f(x)$ be a curve that can be expressed parametrically in the form $x = x(t)$ and $y = y(t)$.
Let the corresponding $t-$values of x_1 and x_2 be t_1 and t_2 respectively.

We need an expression for $\int_{x_1}^{x_2} y dx$ with dx replaced by dt.

$$\begin{aligned} dx &= dx \times \frac{dt}{dt} \\ &= \frac{dx}{dt} dx \end{aligned}$$

Then $\int_{x_1}^{x_2} y dx = \int_{t_1}^{t_2} y \frac{dx}{dt} dt$

N.B. Remember to change the limits as you change from dx to dt
Given the nature of the formula, integration by parts may be required

E.g. 1 Find an expression in parametric form that is equivalent to $\int y dx$:

(a) $x = \frac{3}{t}, y = 4t^2$ (b) $x = \sqrt{t}, y = 3t^2 - 4$

Working: (a) From $x = \frac{3}{t} = 3^{-t}$, $\frac{dx}{dt} = -3t^{-2} = -\frac{3}{t^2}$

$$\int y \frac{dx}{dt} dt = \int 4t^2 \times -\frac{3}{t^2} dt = -\int 12 dt$$

E.g. 2 A curve has parametric equations $x = 3t^2$, $y = \frac{5}{t}$, where $t > 0$. Find the value of $\int_3^{75} y dx$.

E.g. 3 A curve has parametric equations $x = 6t^2$, $y = e^{2t}$, where $t > 0$. Find the value of $\int_0^6 y dx$.

Video A: Integrating parametric functions
Video B: Integrating parametric functions

Solutions to Starter and E.g.s

Exercise
p264 12D Qu 1-4

Summary

$$\int_{x_1}^{x_2} y\,dx = \int_{t_1}^{t_2} y\frac{dx}{dt}\,dt$$