Integrating $\sin^2 x$ and $\cos^2 x$

Starter

1. **(Review of A2 material)** Integrate: (a) $\int \sin 6x\,dx$ (b) $\int 7\cos 3x\,dx$

2. **(Review of A2 material)** State the double angle identities for $\cos 2x$:
   (a) involving $\cos x$ only and (b) involving $\sin x$ only.

3. **(Review of A2 material)**
   State the identity (a) for $\cos 4x$ involving $\sin 2x$ (b) $\cos 10x$ involving $\cos 5x$

4. Express, in terms of cosine and using double angle formulae, (a) $\cos^2 3x$ (b) $\sin^2 7x$

5. Hence find: (a) $\int \cos^2 3x\,dx$ (b) $\int \sin^2 7x\,dx$

Working: (a)
$$\begin{aligned}\int \cos^2 3x\,dx &= \int \frac{1}{2}(1+\cos 6x)\,dx \\ &= \int \left(\frac{1}{2}+\frac{1}{2}\cos 6x\right)dx \\ &= \frac{1}{2}x+\frac{1}{12}\sin 6x + c\end{aligned}$$

Notes

Integrating $\sin^2 Ax$ ***and*** $\cos^2 Ax$

We cannot integrate $\sin^2 x$ and $\cos^2 x$ directly but we can integrate multiple angle formulae easily enough so we express them in terms of $\cos 2x$.

Integrating $\sin^2 Ax$

$$\cos 2x = 1-2\sin^2 x \quad \Rightarrow \quad \sin^2 x = \frac{1}{2}-\frac{1}{2}\cos 2x \quad \therefore \sin^2 Ax = \frac{1}{2}-\frac{1}{2}\cos 2Ax$$

In general: $\int \sin^2 Ax\,dx = \int \left(\frac{1}{2}-\frac{1}{2}\cos 2Ax\right)dx$

Integrating $\cos^2 Ax$

$$\cos 2x = 2\cos^2 x - 1 \quad \Rightarrow \quad \cos^2 x = \frac{1}{2}+\frac{1}{2}\cos 2x \quad \therefore \cos^2 Ax = \frac{1}{2}+\frac{1}{2}\cos 2Ax$$

In general: $\int \cos^2 Ax\,dx = \int \left(\frac{1}{2}+\frac{1}{2}\cos 2Ax\right)dx$

E.g. 1 Find $\int \cos^2 5x\,dx$.

Video: Integration involving sin^2/cos^2
Video: Integration using trigonomeric identities

Integrals involving trigonometric functions EQ

Solutions to Starter and E.g.s

Exercise
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Summary

Using $\sin^2 Ax = \frac{1}{2} - \frac{1}{2}\cos 2Ax$: $\int \sin^2 Ax\,dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2Ax\right)dx$

Using $\cos^2 Ax = \frac{1}{2} + \frac{1}{2}\cos 2Ax$: $\int \cos^2 Ax\,dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2Ax\right)dx$