Integrating sin *x* cos *x*

Starter

1. **(Review of last lesson)** Find: (a) $\int 6\sin^2 x\,dx$ (b) $\int 4+\cot^2 3x\,dx.$

2. **(Review of A2 material)** State the double angle identity for $\sin 2A$.

3. Express $8\sin 6x\cos 6x$ in terms of sine only. Hence find $\int 8\sin 6x\cos 6x\,dx.$

Notes

In general: $\int k\sin Ax\cos Ax\,dx = \frac{k}{2}\int \sin 2Ax\,dx$

E.g. 1 Find: (a) $\int \sin 2x\cos 2x\,dx$ (b) $\int (\sin x+2)^2 dx$

(c) $\int 3\tan^2 4x\,dx$ (d) $\int 6\sin 5x\cos 5x\,dx$

Video: Integration using trigonometric identities

Integrals involving trigonometric functions EQ

Solutions to Starter and E.g.s

Exercise

No questions in textbook

Summary

Using $\sin^2 Ax = \frac{1}{2}-\frac{1}{2}\cos 2Ax$: $\int \sin^2 Ax\,dx = \int\left(\frac{1}{2}-\frac{1}{2}\cos 2Ax\right)dx$

Using $\cos^2 Ax = \frac{1}{2}+\frac{1}{2}\cos 2Ax$: $\int \cos^2 Ax\,dx = \int\left(\frac{1}{2}+\frac{1}{2}\cos 2Ax\right)dx$

Using $1+\tan^2 nx \equiv \sec^2 nx$: $\int \tan^2 x\,dx = \int(\sec^2 x-1)dx = \tan x - x + c$

Using $1+\cot^2 nx \equiv cosec^2 nx$: $\int \cot^2 x\,dx = \int(cosec^2 x-1)dx = -\cot x - x + c$

$$\int k\sin Ax\cos Ax\,dx = \frac{k}{2}\int \sin 2Ax\,dx$$