

Integrating $\tan^2 x$ and $\cot^2 x$

Starter

1. **(Review of A2 material)** Integrate: (a) $\int \sec^2 x dx$ (b) $\int \operatorname{cosec}^2 x dx$

2. State the identity involving $\tan^2 x$ and $\sec^2 x$. Hence find $\int \tan^2 x dx$.

Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\cos^2 x$.

3. State the identity for $\tan^2 3x$ involving \sec . Hence find $\int \tan^2 3x dx$.

Hint: Write $\tan^2 3x = \dots$

4. State the identity involving $\cot^2 x$ and $\operatorname{cosec}^2 x$. Hence find $\int \cot^2 x dx$.

Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\sin^2 x$.

Notes

When integrating $\tan^2 x$ use the identity involving $\tan^2 x$ and $\sec^2 x$: $1 + \tan^2 x \equiv \sec^2 x$

$$\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + c \quad \text{since } 1 + \tan^2 x \equiv \sec^2 x$$

N.B. $1 + \tan^2 nx \equiv \sec^2 nx$

When integrating $\cot^2 x$ use the identity involving $\cot^2 x$ and $\operatorname{cosec}^2 x$: $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

$$\int \cot^2 x dx = \int (\operatorname{cosec}^2 x - 1) dx = -\cot x - x + c \quad \text{since } 1 + \cot^2 x \equiv \operatorname{cosec}^2 x$$

N.B. $1 + \cot^2 nx \equiv \operatorname{cosec}^2 nx$

E.g. 1 Find $\int \tan^2 3x dx$.

E.g. 2 Find $\int \cot^2 7x dx$

Video: [Integration using trigonometric identities](#)

[Integrals involving trigonometric functions EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

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Summary

Using $\sin^2 Ax = \frac{1}{2} - \frac{1}{2} \cos 2Ax$: $\int \sin^2 Ax dx = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2Ax \right) dx$

Using $\cos^2 Ax = \frac{1}{2} + \frac{1}{2} \cos 2Ax$: $\int \cos^2 Ax dx = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2Ax \right) dx$

Using $1 + \tan^2 nx \equiv \sec^2 nx$: $\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + c$

Using $1 + \cot^2 nx \equiv \operatorname{cosec}^2 nx$: $\int \cot^2 x dx = \int (\operatorname{cosec}^2 x - 1) dx = -\cot x - x + c$