Integrating $\tan^2 x$ and $\cot^2 x$

Starter

1. **(Review of A2 material)** Integrate: (a) $\int \sec^2 x\,dx$ (b) $\int cosec^2 x\,dx$

2. State the identity involving $\tan^2 x$ and $\sec^2 x$. Hence find $\int \tan^2 x\,dx$.

   Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\cos^2 x$.

3. State the identity for $\tan^2 3x$ involving sec. Hence find $\int \tan^2 3x\,dx$.

   Hint: Write $\tan^2 3x = \ldots$

4. State the identity involving $\cot^2 x$ and $cosec^2 x$. Hence find $\int \cot^2 x\,dx$.

   Hint: Start from $\cos^2 x + \sin^2 x \equiv 1$ and divide each term by $\sin^2 x$.

Notes

When integrating $\tan^2 x$ use the identity involving $\tan^2 x$ and $\sec^2 x$: $1 + \tan^2 x \equiv \sec^2 x$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)dx = \tan x - x + c \qquad \textit{since } 1 + \tan^2 x \equiv \sec^2 x$$

N.B. $1 + \tan^2 nx \equiv \sec^2 nx$

When integrating $\cot^2 x$ use the identity involving $\cot^2 x$ and $cosec^2 x$: $1 + \cot^2 x \equiv cosec^2 x$

$$\int \cot^2 x\,dx = \int (cosec^2 x - 1)dx = -\cot x - x + c \qquad \textit{since } 1 + \cot^2 x \equiv cosec^2 x$$

N.B. $1 + \cot^2 nx \equiv cosec^2 nx$

E.g. 1 Find $\int \tan^2 4x\,dx$.

E.g. 2 Find $\int \cot^2 7x\,dx$

Video: Integration using trigonometric identities

Integrals involving trigonometric functions EQ

Solutions to Starter and E.g.s

Exercise
p238 11F Qu 3b(i&ii), 4b(i&ii), 8

Summary

Using $\sin^2 Ax = \frac{1}{2} - \frac{1}{2}\cos 2Ax$: $\int \sin^2 Ax\,dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2Ax\right)dx$

Using $\cos^2 Ax = \frac{1}{2} + \frac{1}{2}\cos 2Ax$: $\int \cos^2 Ax\,dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2Ax\right)dx$

Using $1 + \tan^2 nx \equiv \sec^2 nx$: $\int \tan^2 x\,dx = \int (\sec^2 x - 1)dx = \tan x - x + c$

Using $1 + \cot^2 nx \equiv cosec^2 nx$: $\int \cot^2 x\,dx = \int (cosec^2 x - 1)dx = -\cot x - x + c$