

Integration of e^x and $\frac{1}{x}$

Starter

1. Given that $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\ln x)}{dx} = \frac{1}{x}$, find:

(a) $\int e^x dx$ (b) $\int \frac{1}{x} dx$ (c) $\int A e^{kx} dx$ (d) $\int \frac{P}{Qx} dx$

N.B. Remember to include the constant of integration.
Always check your answer by differentiating back.

Notes

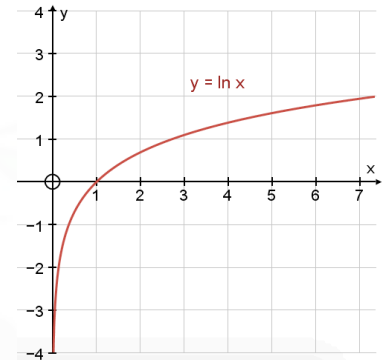
Why do we need the modulus brackets for $\int \frac{1}{x} dx = \ln |x| + c$?

We can only take \ln of positive numbers (see graph).

In general,

$$\int A e^{kx} dx = \frac{A}{k} e^{kx} + c$$

$$\int \frac{P}{Qx} dx = \frac{P}{Q} \int \frac{1}{x} dx = \frac{P}{Q} \ln |x| + c$$



E.g. 1 Find: (a) $\int 8e^{3x} dx$ (b) $\int \frac{1}{9} e^{4x} dx$ (c) $\int \frac{1}{8x} dx$ (d) $\int \frac{3}{7x} dx$

Working: (a) $\frac{8}{3} e^{3x} + c$

E.g. 2 Find the equation of the curve that has derivative $5e^{4x}$ given that the curve passes through the point $(2, 6e^8)$. Give your answer exactly.

Working: The derivative is derivative $5e^{4x}$ so $\frac{dy}{dx} = 5e^{4x}$
 Integrate to find y : $y = \int 5e^{4x} dx = \frac{5}{4} e^{4x} + c$
 To find c , use $(2, 6e^8)$: $6e^8 = \frac{5}{4} e^{16} + c$
 $c = \frac{19}{4} e^8$
 So the curve is $y = \frac{5}{4} e^{4x} + \frac{19}{4} e^8$

E.g. 3 Find the value of $\int_2^3 e^{2x} dx$.

E.g. 4 Express $\int_3^{15} \frac{2}{x} dx$ in the form $\ln A$ where A is to be found.

Video: [Integrating e^x](#)

Video: [Integrating reciprocal functions](#)

[Solutions to Starter and E.g.s](#)

Exercise

p193 9B Qu 1ia-c, 2i, 3ia-e, 4, 7, 8, 11, 13, 14

Summary

$$\int A e^{kx} dx = \frac{A}{k} e^{kx} + c$$

$$\int \frac{P}{Qx} dx = \frac{P}{Q} \int \frac{1}{x} dx = \frac{P}{Q} \ln|x| + c$$