

## Integration using Partial Fractions

### Starter

1. **(Review of last lesson)** Find: (a)  $\int_0^{\frac{\pi}{4}} \cos^2 4x dx$  (b)  $\int_0^{\frac{\pi}{4}} \cos 2x \sin 2x dx$

2. **(Review of A2 material)** (a) Express  $\frac{3x - 2}{x^2 + x - 12}$  as partial fractions.

(b) Hence find  $\int \frac{3x - 2}{x^2 + x - 12} dx$ , expressing your answer as a single logarithm.

3. **(Review of AS material)**

Find the quotient and remainder when you divide  $4x^3 - 2x^2 + 6$  by  $x + 3$ .

### Notes

Review of partial fractions

1. Deg Top < Deg Bottom:  $\frac{A}{(\quad)} + \frac{B}{(\quad)}$

2. Deg Top < Deg Bottom (repeated):  $\frac{A}{(\quad)} + \frac{B}{(\quad)} + \frac{C}{(\quad)^2}$

**E.g.**  $\frac{3x + 2}{(x + 5)(x - 3)^2} \equiv \frac{A}{x + 5} + \frac{B}{x - 3} + \frac{C}{(x - 3)^2}$

3. Deg Top = Deg Bottom:  $A + \frac{B}{(\quad)} + \frac{C}{(\quad)}$

**E.g.**  $\frac{x^2 + 4x - 1}{(x - 7)(x + 2)} \equiv A + \frac{B}{x - 7} + \frac{C}{x + 2}$

4. Deg Top = Deg Bottom + 1:  $Ax + B + \frac{C}{(\quad)} + \frac{D}{(\quad)}$

**E.g.**  $\frac{x^3 + 4x - 1}{(x - 2)(x + 3)} \equiv Ax + B + \frac{C}{x - 2} + \frac{D}{(x + 3)}$

- E.g. 1** (a) Express  $\frac{11x - 10}{x(x - 5)^2}$  as partial fractions.  
 (b) Hence find  $\int \frac{11x - 10}{x(x - 5)^2} dx$

**Working:** (b) 
$$\begin{aligned} \int \frac{11x - 10}{x(x - 5)^2} dx &= \int -\frac{2}{5x} + \frac{2}{5(x - 5)} + \frac{9}{(x - 5)^2} dx \\ &= \int -\frac{2}{5x} + \frac{2}{5(x - 5)} + 9(x - 5)^{-2} dx \\ &= -\frac{2}{5} \ln|x| + \frac{2}{5} \ln|x - 5| - 9(x - 5)^{-1} + c \\ &= \frac{2}{5} \ln \left| \frac{x - 5}{x} \right| - \frac{9}{(x - 5)} + c \quad \text{2nd law of logs} \end{aligned}$$

- E.g. 2** Evaluate  $\int_0^1 \frac{x}{(x - 2)(x - 3)} dx$ , expressing your answer as a single logarithm.

[Video: Integrals involving partial fractions](#)

[Integrals involving partial fractions EQ](#)

[Video: Integration methods](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p242 11G Qu 3i, 4i, 5i, 6-9

**Summary**

- Deg Top < Deg Bottom:  $\frac{A}{(\quad)} + \frac{B}{(\quad)}$
- Deg Top < Deg Bottom (repeated):  $\frac{A}{(\quad)} + \frac{B}{(\quad)} + \frac{C}{(\quad)^2}$   
**E.g.**  $\frac{3x + 2}{(x + 5)(x - 3)^2} \equiv \frac{A}{x + 5} + \frac{B}{x - 3} + \frac{C}{(x - 3)^2}$
- Deg Top = Deg Bottom:  $A + \frac{B}{(\quad)} + \frac{C}{(\quad)}$   
**E.g.**  $\frac{x^2 + 4x - 1}{(x - 7)(x + 2)} \equiv A + \frac{B}{x - 7} + \frac{C}{x + 2}$
- Deg Top = Deg Bottom + 1:  $Ax + B + \frac{C}{(\quad)} + \frac{D}{(\quad)}$   
**E.g.**  $\frac{x^3 + 4x - 1}{(x - 2)(x + 3)} \equiv Ax + B + \frac{C}{x - 2} + \frac{D}{(x + 3)}$