Inverse Normal distribution

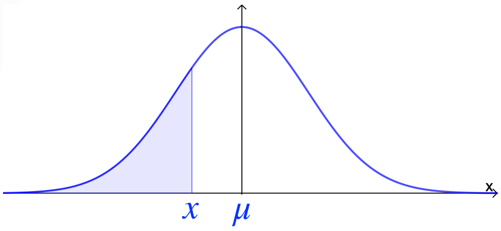
Starter

1. The heights of a group of people are normally distributed with a mean of 174 cm and a standard deviation of 6 cm. Find the probability that a person selected at random:
   (a) is at least 170 cm tall
   (b) is no taller than 180 cm
   (c) is at least 178 cm given that they are between 172 and 182 cm.

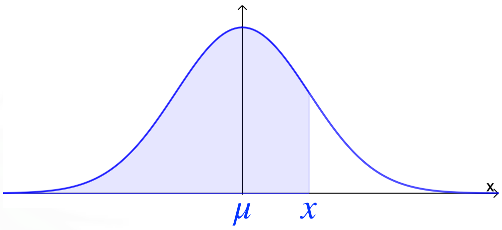
Notes

So far questions have given the $x-$values and we have had to calculate the probabilities i.e. the area under the normal curve between the set values.

In this lesson, we will be asked to calculate the $x-$value for a given area.



Once again modern calculators facilitate the calculation but a little thinking is required because they generally only give the $x-$value when the area from $-\infty$ to x i.e. $P(X < x)$. See the diagrams to the right.



Therefore, if we are asked to find the $x-$value such that $P(X > x) = \ldots$ we must first convert it into a form involving $P(X < x)$.

Calculating $x-$ values using a calculator

In the past, statisticians used tables of data to calculate probabilities but nowadays a modern calculator can do it.

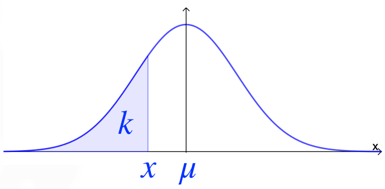
On the Classwiz: Menu >> 7:Distribution >> 3:Inverse Normal

Input screen **Inverse normal**

Area: **type a large number between 0 and 1**
σ: **square root to get standard deviation if $X \sim$ N$(5, 7)$**
μ:

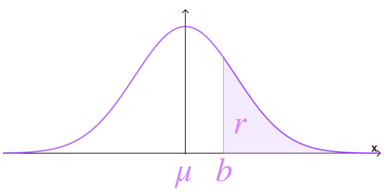
Considering areas diagrammatically

E.g. 1 The inverse normal function of a modern calculator calculates the $x-$value when the area is similar to the one in the diagram $P(X < x) = k$.



(a) Sketch a diagram and shade the area represented by r for $P(X > b) = r$. Write down a formula for $P(X < b)$ involving r.
(b) Sketch a diagram and shade the area represented by s for $P(|X - \mu| < b) = s$. Write down a formula for $P(X < \mu + b)$ involving s, but not $P(X < \mu - b)$.
(c) Sketch a diagram and shade the area represented by t for $P(a < X < b) = t$, where $a < \mu < b$. If $P(X < a)$ is given, find a formula for $P(X < b)$ in terms of t and $P(X < a)$.

Working: (a) $P(X > b) = r$
$P(X < b) = 1 - r$
since area under normal curve is 1



E.g. 2 Given that $X \sim \text{N}(52, 40)$, find p, q, r and s correct to 3 s.f. such that:

(a) $P(X < p) = 0.647$ (b) $P(X > q) = 0.581$

(c) $P(52 < X < r) = 0.3$ (d) $P(|X - 52| < s) = 0.36$

Working: (a) $P(X < p) = 0.647 \Rightarrow p = 53.4$ (3 s.f.)

E.g. 3 Given that $y \sim \text{N}(20, 2^2)$, find a, b, c and d correct to 3 s.f. such that:

(a) $P(X > a) = 0.28$ (b) $P(X < b) = 0.183$

(c) $P(|X - 20| < c) = 0.84$ (d) $P(18 < X < d) = 0.45$

Working: (a) $P(X > a) = 0.28 \Rightarrow P(X < a) = 1 - 0.28 = 0.72$

$\therefore q = 21.2$ (3 s.f.)

E.g. 4 The board of examiners have decided that $80\,\%$ of all candidates sitting A level maths will obtain a pass grade. The actual exam marks are found to be normally distributed with a mean of 45 and a standard deviation of 7.

(a) What is the lowest score a student can get on the exam to be awarded a pass grade?

(b) Given that $10\,\%$ of students will achieve an A*, calculate the lowest mark required to get the highest grade.

Give your answers to the nearest mark.

E.g. 5 Batteries for a radio have a mean life of 160 hours and a standard deviation of 30 hours. Assuming the battery life follows a normal distribution, calculate:

(a) the proportion of batteries which have a life between 150 and 180 hours

(b) the range, symmetrical about the mean, within which $75\,\%$ of batteries lie.

Video: Inverse normal function

Normal distribution EQ

Solutions to Starter and E.g.s

Exercise

p386 17C Qu 1i, 2-7, (8-11 red)

Summary

Calculating $x-$values using a calculator:

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μ:

1. $P(X > x) = k \Rightarrow P(X < x) = 1 - k$ ***since area under normal curve is 1***
2. $P(|X - \mu| < a) \equiv P(\mu - a < X < \mu + a) = k$

   $\Rightarrow P(\mu < X < \mu + a) = \dfrac{k}{2}$ ***symmetry***

   $\Rightarrow P(X < \mu + a) = \dfrac{k}{2} + 0.5$ ***since $P(X < \mu) = 0.5$***
3. $P(p < X < q) = k$, where p is given $\Rightarrow P(X < q) = k + P(X < p)$