Modelling Projectile Motion

Starter

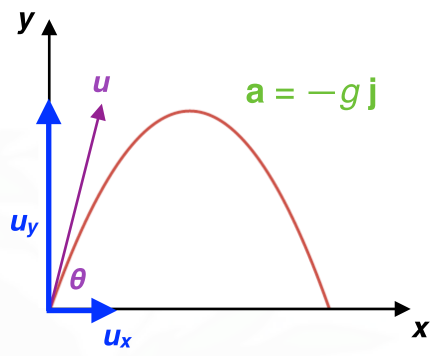
1. (Review of AS material) A particle is projected vertically upward from ground level with a speed of 24.5 m/s. Find:
   (a) the greatest height reached
   (b) the time that elapses before the particle returns to the ground.

2. (Review of AS material) A particle starts from the origin and moves under acceleration $\mathbf{a} = (2\mathbf{i} - 7\mathbf{j})$ m/s². Given that the initial velocity is $\mathbf{u} = (-\mathbf{i} + 6\mathbf{j})$. Find:
   (a) the position and velocity of the particle after 4 s and
   (b) the time when the particle is at the point $(\lambda\mathbf{i} - 12\mathbf{j})$, where λ is a value to be found.

Notes

At AS level, we looked at objects thrown straight up in the air. Now we will look at objects thrown at an angle.

Consider an object projected with initial velocity u and at an angle of θ to the horizontal.



By modelling the object as a particle and ignoring air resistance, we assume the only force acting on the object is gravity i.e. $\mathbf{a} = -g\mathbf{j}$. The ***trajectory***, or path, of the projectile is a ***parabola***.

u_x and u_y are the components of the initial velocity in the $x-$ and $y-$directions respectively.

Using trigonometry: $\cos\theta = \dfrac{u_x}{u}$ $\quad\sin\theta = \dfrac{u_y}{u}$

So initial velocities: $u_x = u\cos\theta$ $\quad u_y = u\sin\theta$

E.g. 1 Find the horizontal and vertical components of the velocity when a particle is moving with speed 8 m/s at an angle of 35° above the horizontal.

E.g. 2 A ball is thrown with velocity v, with horizontal component 6 m/s and vertical component 8 m/s. Find the speed and direction of projection of the ball.

N.B. Since there is ***no horizontal force*** acting, the ***horizontal*** component of the ***velocity*** is ***constant*** throughout the motion.

As acceleration is constant, the vector form of the SUVAT equations can be used:

Scalar form	Vector form	Unknown not included
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	No $\mathbf{s}$
$s = \dfrac{1}{2}(u+v)t$	$\mathbf{s} = \dfrac{1}{2}(\mathbf{u}+\mathbf{v})t$	No $\mathbf{a}$
$s = ut + \dfrac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \dfrac{1}{2}\mathbf{a}t^2$	No $\mathbf{v}$

$s = vt - \frac{1}{2}at^2$ $\qquad$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $\qquad$ No $\mathbf{u}$

$v^2 = u^2 + 2as$ $\qquad$ No t

N.B. When using the scalar form, you need to state 'horizontally' or 'vertically' ***or*** use $u_x = u\cos\theta$ and $u_y = u\sin\theta$.

E.g. 3 A particle is projected with initial speed 18 m/s at an angle of 40^o above the horizontal. Find the distance from its point of projection 1.8 s after it is projected.

Working: ***The aim is to find the distance travelled horizontally and vertically and then use Pythagoras.***

E.g. 4 A particle is projected from the point $5\mathbf{j}$. The particle's initial velocity is $(7\mathbf{i} + 3\mathbf{j})$ m/s and it moves freely under gravity.
(a) Find its position vector 0.8 s after it is projected.
(b) Find the velocity at this time.

Working: (a) $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j}, t = 0.8, \mathbf{a} = -9.8\mathbf{j}, \mathbf{s} = ?$

No $\mathbf{v}$ $\Rightarrow$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ …

(b)

Video: Projectiles
Video: Projectiles (upward angle/above ground)
Video: Projectiles (downward angle/above ground)
Video: Projected horizontally

Solutions to Starter and E.g.s

Exercise
p455 20A Qu 1-4

Additional questions

1. Find the horizontal and vertical components of the velocity when a particle is…
   (a) …fired vertically upwards with speed 45 m/s
   (b) …fired at an angle of α above the horizontal with speed 22 km/h

2. A stone is thrown horizontally with speed 10 m/s from a height of 2 m above horizontal ground.
   (a) Find the speed and direction of motion of the stone after 0.5 s.
   (b) Find the stone's horizontal displacement when it hits the ground.

3. A particle is projected with velocity $(12\mathbf{i} + 16\mathbf{j})$ m/s. Find the particle's velocity:
   (a) at 2 s after projection
   (b) when it reaches its maximum height
   (c) when it hits the ground.

4. A particle is fired 5 m above ground with initial velocity $(17\mathbf{i} + 10\mathbf{j})$ m/s. Find:
   (a) the particle's maximum height
   (b) the speed of the particle as it hits the ground
   (c) the direction of the particle when it hits the ground.

5. A stone is thrown with velocity $(6\mathbf{i} + 9\mathbf{j})$ m/s, from a window 2.5 m above the ground, towards a target which is a horizontal distance of 20 m away on the ground.
Find:
(a) the length of time the stone is at least 6 m above the ground.
(b) the distance by which the stone falls short of the target.

6. A golf ball is hit off the edge of a 40 m high vertical cliff with velocity $(a\mathbf{i} + b\mathbf{j})$ m/s, where a and b are constants. It takes 5 s to land on the ground below, level with the foot of the cliff, a horizontal distance of 200 m away. Find:
(a) the values of a and b
(b) the velocity of the golf ball when it hits the ground.

Summary

The ***trajectory***, or path, of the projectile is a ***parabola***.

The only force acting on the object is gravity i.e. $\mathbf{a} = -g\mathbf{j}$.

u_x and u_y are the components of the initial velocity: $u_x = u\cos\theta$ $u_y = u\sin\theta$

Since there is ***no horizontal force*** acting, the ***horizontal*** component of the ***velocity*** is ***constant*** throughout the motion.

SUVAT equations apply:

Scalar form	Vector form	Unknown not included
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	No $\mathbf{s}$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$	No $\mathbf{a}$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	No $\mathbf{v}$
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$	No $\mathbf{u}$
$v^2 = u^2 + 2as$		No t