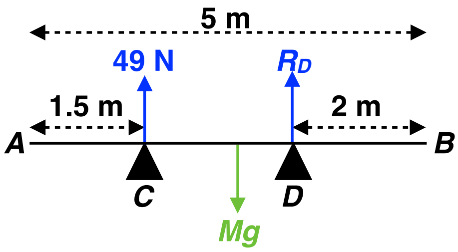
Non-uniform rods, tilting and moment of angled force

Starter

1. (Review of last lesson)
A uniform rod, AB, of length 5 m and mass M kg, rests horizontally in equilibrium on supports at C and D, as shown. If the magnitude of the normal reaction at C is 49 N, find:
(a) the magnitude of the normal reaction at D,
(b) the mass, M, of the rod.



Notes

Non-uniform rods

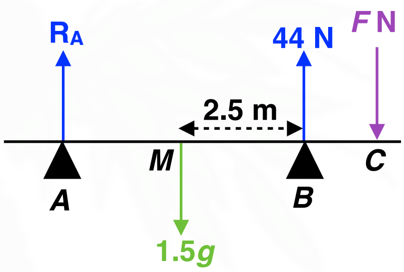
A non-uniform rod means that the weight does not act at the centre of the rod and could be at any point along the rod.

E.g. 1 A non-uniform beam, AB, of mass 3 kg and length 1.6 m rests horizontally in equilibrium on vertical supports at A and B. The normal reaction at A is 12 N. Find
(a) the magnitude of the normal reaction at B
(b) the distance of the centre of mass of the beam from A.

Tilting

If a rod is about to tilt about a point, then any ***normal reaction*** acting at ***any other support*** along the rod will be ***zero***.

E.g. 2 A non-uniform rod of mass 1.5 kg rests horizontally in equilibrium on supports at A and B, as shown. When a downwards force of magnitude F N is applied to the rod at C, the rod remains horizontal and in equilibrium, but is on the point of tilting about B, where the normal reaction has magnitude 44 N. Given that the distance between B and the centre of mass is 2.5 m, find:
(a) the value of F,
(b) the distance between the points B and C.



Working: (a) The rod is on the point of tilting about B so the normal reaction at A is zero i.e. $R_A = 0$

$R(\uparrow)$: $F + 1.5g = 44$

$F = 44 - 1.5g$

$F = 29.3$ N

(b) Let d be the distance between B and C

$\circlearrowright$ about B: $29.3 \times d = 1.5g \times 2.5$

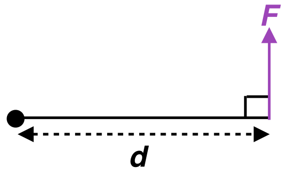
$$d = \frac{735}{586} \approx 1.25$$

The distance between the points B and C is 1.25 m (3 s.f.)

Turning effect of non-perpendicular forces

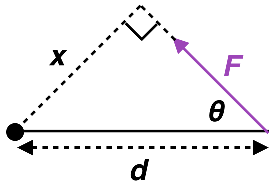
Moment $= F \times d$ where d is the perpendicular force.

Perpendicular force



Moment $= F \times d$

Non-perpendicular force

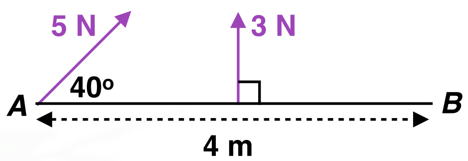


Perpendicular distance, $x = d \sin \theta$

Moment $= F \times d \sin \theta$

However, $F \times d \sin \theta = F \sin \theta \times d$ so instead of finding the perpendicular distance, we could simply resolve the ***force in the perpendicular direction***.

E.g. 3 A light rod, AB, has length 4 m. A force of magnitude 5 N is applied to A at an angle of 40° above the rod, as shown. A second force of magnitude 3 N is applied vertically upwards at the rod's midpoint. Find the sum of the moments of the forces acting on the rod about B.



Working: Resolve the 5 N force perpendicular to the rod: $5 \sin 40$

↻ about B: $5 \sin 40 \times 4 + 3 \times 2 \approx 18.9$

The sum of the moments about B is 18.9 Nm (3 s.f.) clockwise.

Video: Tilting

Video: Moment of non-perpendicular force

Tilting EQ

Moment of non-perpendicular force EQ

Solutions to Starter and E.g.s

Exercise

p517 22C Qu 1-7

Summary

Non-uniform rods — the weight does not act at the centre of the rod.

Tilting

If a rod is about to tilt about a point, then any ***normal reaction*** acting at ***any other support*** along the rod will be ***zero***.

Turning effect of non-perpendicular forces

Moment $= F \sin \theta \times d$ resolve the ***force in the perpendicular direction***.