Parametric Equations

Starter

1. **(Review of last lesson)** Decide whether the curve $y = x^2 - \frac{1}{x}$ has a point of inflexion.

2. **(Review of last lesson)**
If $f(x) = \frac{1}{16}x^4 + \frac{3}{4}x^3 - \frac{21}{8}x^2 - 6x + 20$, identify the range of values of x for which the graph of $y = f(x)$ is concave and convex. Express your answers in set notation.

3. Find an expression involving y and x but no t given that:
   (a) $x = 3t$ and $y = \frac{6}{t}$
   (b) $x = 4t^2$ and $y = 8t$

Notes

Cartesian form: y in terms of x i.e. explicit $y = f(x)$ or implicit $f(x, y) = k$
E.g. $y = x^2, x^2 + y^2 = 4$

Parametric form: x and y in terms of another letter (the parameter) i.e. $x = f(t), y = g(t)$
E.g. $x = t, y = t^2$

N.B. The curve $y = x^2$ is the same as $x = t$ and $y = t^2$

Transforming from parametric to cartesian form
Either rearrange one parametric equation for t and substitute in the other equation, ***or***:

Rearrange both equations so that they equal the same expression of t and then equate and rearrange.

N.B. When involving trigonometry, you will normally need to use a trigonometric identity.

E.g. 1 Express $x = 3\cos t, y = 4\sin t$ in cartesian form.

Working:
$$\cos^2 t + \sin^2 t = 1$$
$$\text{so } \left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$
$$16x^2 + 9y^2 = 144$$

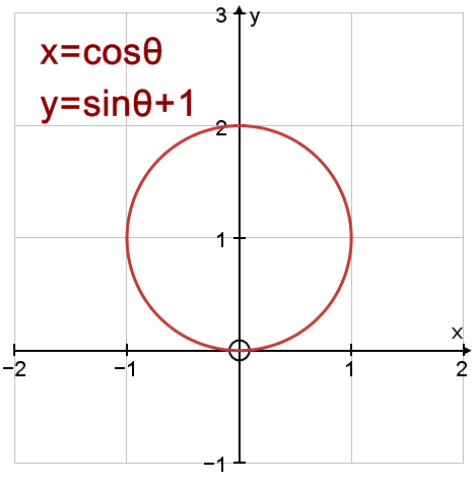
Reminder of trigonometric identities
$\cos 2t = \cos^2 t - \sin^2 t$ $\quad$ $\cos 2t = 2\cos^2 t - 1$ $\quad$ $\cos 2t = 1 - 2\sin^2 t$
$\sin 2t = 2\sin t \cos t$
$1 + \tan^2 t = \sec^2 t$

Sketching curves from parametric equations
Either make a table of values or convert into cartesian form and sketch.

E.g. 2 Sketch the graph given by $x = \cos\theta$ and $y = \sin\theta + 1$.

ϑ	0	π/2	π	3π/2	2π
x	1	0	−1	0	1
y	1	2	1	0	1

Working: Circle, centre (0, 1) and radius 1



E.g. 3 Transform these parametric curves into cartesian form:

(a) $x = \dfrac{5}{2-t}, \quad y = \dfrac{4}{2-t}$

(b) $x = \cos t, \quad y = 4\sin 2t \sin t$

E.g. 4 The curve C is defined by the parametric equations $x = 2t^2 - 7t, \ y = 10 - t^2$.

(a) Find the value of a if $(a, \ 1)$ is a point on the curve and $a > 1$.

(b) Decide whether $(-6, 4)$ lies on curve C.

Video: Introduction to parametric functions

Parametric functions EQ

Solutions to Starter and E.g.s

Exercise
p256 12B Qu 2i, 3iac, 4i, 5-7

Summary
Transforming from parametric to cartesian form:

Either rearrange one parametric equation for t and substitute into the other equation,
or rearrange both equations so that they equal the same expression of t and then equate and rearrange,
or use a trigonometric identity.