

## Quotient Rule

### Starter

To differentiate functions of the form  $y = uv$  we use the product rule:  $\frac{dy}{dx} = u'v + v'u$

### Poetry

Differentiate first term, times by second,  
Add,  
Differentiate second term, times by first.

- (Review of last lesson)** Differentiate  $y = e^{-2x} \cos 5x$ .
- We use the quotient rule to differentiate when we have one function divided by another i.e.  $y = \frac{u}{v}$  where  $u$  and  $v$  are functions of  $x$ .  
How could we change the function  $y = \frac{u}{v}$  into a form where we could use the product rule?
- Hence using the product rule, and possibly the chain rule, find  $\frac{dy}{dx}$  when  $y = \frac{u}{v}$

### Notes

The quotient rule is given by

$$\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$$

### Poetry

Differentiate top, times by bottom,  $u'v$   
Subtract,  $u'v -$   
Differentiate bottom, times by top,  $u'v - v'u$   
All over bottom squared.  $\frac{u'v - v'u}{v^2}$

(Hamlet, Act V, Scene 1, Gravedigger scene or "Alas, Poor Yorick scene")


**E.g. 1** Find the derivative of these functions:

(a)  $y = \frac{3x}{x^2 + 7}$

(b)  $y = \frac{e^{6x}}{x^2}$

(c)  $y = \frac{\sin 4x}{5x^3}$

**Working:** (a)  $u = 3x$   $u' = 3$   
 $v = x^2 + 7$   $v' = 2x$



The arrows can be useful but you need to get the order right in the numerator as we are subtracting (Differentiate top, times by bottom)

$$\begin{aligned} \frac{dy}{dx} &= \frac{3(x^2 + 7) - 2x \times 3x}{(x^2 + 7)^2} \\ \frac{dy}{dx} &= \frac{3x^2 + 21 - 6x^2}{(x^2 + 7)^2} \\ \frac{dy}{dx} &= \frac{3(7 - x^2)}{(x^2 + 7)^2} \end{aligned}$$

**E.g. 2** Find the coordinates of the stationary point(s) of the curve  $y = \frac{5x - 4}{2x^2}$  and determine its/ their nature.

- E.g. 3** (a) By writing  $y = \sec x$  as  $y = \frac{1}{\cos x}$  and using the quotient rule, find  $\frac{dy}{dx}$ .
- (b) Using a similar method, find  $\frac{dy}{dx}$  when:
- (i)  $y = \operatorname{cosec} x$
  - (ii)  $y = \cot x$

**Video:** [Quotient rule](#)

[Differentiation methods EQ](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p208 10C Qu 1i, 2-7

### Summary

Product rule:  $\frac{dy}{dx} = \frac{du}{dx}v + \frac{dv}{dx}u$  or  $\frac{dy}{dx} = u'v + v'u$

Poetry: Differentiate first term,  
Times by second,  
Add,  
Differentiate second term,  
Times by first.

Quotient rule:  $\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$

Poetry: Differentiate top, times by bottom,  
Subtract,  
Differentiate bottom, times by top,  
All over bottom squared.