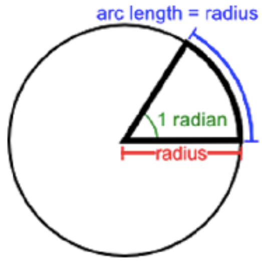
Radians

Starter

Radians is a different unit of measurement for angles. It is important because all calculus with trigonometric functions is done with radians.



Definition

One radian is defined as the angle subtended at the centre of the circle such that the arc length is the same length as the radius.

N.B. Arc length is usually given the letter, s.

1. How many degrees are equivalent to one radian?

   Hint: Arc length, $s = \dfrac{\theta}{360^{\circ}} \times 2\pi r$

Notes

How do we convert from radians to degrees? $\times \dfrac{180}{\pi}$

How do we convert from degrees to radians? $\times \dfrac{\pi}{180}$

$$\text{Degrees} \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \text{Radians}$$

Notation

Radians, when given as decimals, are denoted by a ^{c}: 2.63 radians $\equiv 2.63^c$.

However, when an angle is given in terms of π, we do not include the ^{c}.

E.g. 1 Convert 45° to radians.

E.g. 2 Convert the following to degrees: (a) $\dfrac{2\pi}{3}$ (b) 1.85^c

Common angles

E.g. 3 Copy and complete this table.

Degress	0°	30°	45°		90°	120°		150°	180°		360°
Radians	0^c		$\frac{\pi}{4}$	$\frac{\pi}{3}$		$\frac{2\pi}{3}$	$\frac{3\pi}{4}$		π	$\frac{3\pi}{2}$	2π

Identities from AS level maths converted to radians

Periodic behaviour

$\sin(\theta \pm 2\pi) = \sin\theta$ (i.e. the graph repeats every 2π)

E.g. $\sin\frac{9\pi}{4} = \sin\left(\frac{\pi}{4} + 2\pi\right) = \sin\frac{\pi}{4}$

$\cos(\theta \pm 2\pi) = \cos\theta$ (i.e. the graph repeats every 2π)

E.g. $\cos -\frac{\pi}{4} = \cos\left(-\frac{\pi}{4} + 2\pi\right) = \cos\frac{7\pi}{4}$

$\tan(\theta \pm \pi) = \tan\theta$ (i.e. the graph repeats every π)

E.g. $\tan\frac{11\pi}{9} = \tan\left(\frac{2\pi}{9} + \pi\right) = \tan\frac{2\pi}{9}$

N.B. For sin and cos we can replace the 2 by any even number

E.g. $\sin(\theta \pm 6\pi) = \sin\theta$

Symmetry in $y-$axis

Rotational $\sin(-\theta) = -\sin\theta$ ***E.g.*** $\sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4}$

Reflective $\cos(-\theta) = \cos\theta$ ***E.g.*** $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

Combining the above

$\sin(2\pi - \theta) = -\sin\theta$ ***E.g.*** $\sin\frac{15\pi}{8} = \sin\left(2\pi - \frac{\pi}{8}\right) = -\sin\frac{\pi}{8}$

$\cos(2\pi - \theta) = \cos\theta$ ***E.g.*** $\cos\frac{11\pi}{7} = \cos\left(2\pi - \frac{3\pi}{7}\right) = \cos\frac{3\pi}{7}$

Symmetry in the line $x = \frac{\pi}{2}$

$\sin(\pi - \theta) = \sin\theta$ ***E.g.*** Radians: $\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\frac{\pi}{3}$

$\cos(\pi - \theta) = -\cos\theta$ ***E.g.*** Radians: $\cos\frac{2\pi}{3} = -\cos\left(\pi - \frac{2\pi}{3}\right) = -\cos\frac{\pi}{3}$

E.g. 4 Given that $\sin\frac{\pi}{6} = \frac{1}{2}$ state the values of the following without using a calculator:

(a) $\sin\frac{13\pi}{6}$ (b) $\sin\left(-\frac{\pi}{6}\right)$

(c) $\sin\frac{37\pi}{6}$ (d) $\sin\frac{11\pi}{6}$

Working: (a) Periodic: $\sin\frac{13\pi}{6} = \sin\left(\frac{\pi}{6} + 2\pi\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

E.g. 5 Given that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ state the values of the following without using a calculator:

(a) $\cos\frac{25\pi}{6}$ (b) $\cos\left(-\frac{\pi}{6}\right)$

(c) $\cos\left(-\frac{11\pi}{6}\right)$ (d) $\cos\frac{7\pi}{6}$

Working: (a) $\cos\frac{25\pi}{6} = \cos\left(\frac{25\pi}{6} - 4\pi\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Exact values — a reminder

Angle (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Video: Radians

Solutions to Starter and E.g.s

Exercise
p133 7A Qu 2i, 3i, 4i, 6-12

Summary

$$\textbf{Degrees} \quad \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \quad \textbf{Radians}$$

Notation: when given as decimals, radians are denoted by a ^{c}: 2.63 radians $\equiv 2.63^c$.
However, when an angle is given in terms of π, we do not include the ^{c}.

Periodic behaviour

$\sin(\theta \pm 2\pi) = \sin\theta$ $\cos(\theta \pm 2\pi) = \cos\theta$ $\tan(\theta \pm \pi) = \tan\theta$

Symmetry in $y-$axis

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

$\sin(2\pi - \theta) = -\sin\theta$ $\cos(2\pi - \theta) = \cos\theta$

Symmetry in the line $x = \frac{\pi}{2}$

$\sin(\pi - \theta) = \sin\theta$ $\cos(\pi - \theta) = -\cos\theta$