Range and Maximum Height

Starter

1. **(Review of last lesson)** A particle is projected from the ground with initial speed 8 m/s at an angle of 30° above the horizontal.
   (a) Find the time it takes to reach its highest point.
   (b) Find its maximum height.
   (c) Find the time it takes to hit the ground again.
   (d) Find the horizontal displacement of the particle when it hits the ground again.

Notes

Maximum height, s_y

The maximum height of a projectile occurs when the ***vertical velocity is zero*** i.e. $v_y = 0$.

1. Use $v^2 = u^2 + 2as$ vertically

or

1. Use $v = u + at$ (vertically) to find the time taken to get to the maximum height.
2. Substitute the $t-$value from 1. into $s = ut + \frac{1}{2}at^2$ (vertically)

N.B. $u_y = u \sin\theta$

Range, s_x

The range is the ***horizontal*** distance when the projectile ***hits the ground***.

- If projected ***from the ground***, this is when $s_y = 0$
- If projected ***from a balcony or cliff***, this is when $s_y = -h$ (where h is the initial height)

1. Use $s = ut + \frac{1}{2}at^2$ (vertically), put it equal to zero and solve for t to find the time the object is in the air.
2. Use the $t-$value from 1. and substitute it into $s = ut + \frac{1}{2}at^2$ (horizontally), remembering that $a_x = 0$.

N.B. Remember $u_x = u\cos\theta$ and $u_y = u\sin\theta$

E.g. 1 A projectile is launched from the ground with a speed of 15 m/s at an angle of 50° above the horizontal. Find:

(a) the time taken for the projectile to reach its maximum height

(b) the maximum height the projectile reaches above the ground.

Working: (a) Maximum height is when $v_y = 0$

$u_y = 15\sin 50,\ a_y = -9.8,\ v_y = 0,\ t = ?$

No $s_y \quad \Rightarrow \quad v = u + at$: $\quad 0 = 15\sin 50 - 9.8t$

$$t = \frac{15\sin 50}{9.8} = 1.17$$

The time taken to reach its maximum height is 1.17 s (3 s.f.)

(b) Maximum height is when $v_y = 0$

$u_y = 15\sin 50,\ a_y = -9.8,\ v_y = 0,\ s_y = ?$

No $t \quad \Rightarrow \quad v^2 = u^2 + 2as$: $\quad 0^2 = (15\sin 50)^2 - 2 \times 9.8 \times s_y$

$$s_y = \frac{(15\sin 50)^2}{2 \times 9.8} = 6.74$$

The maximum height is 6.74 m

E.g. 2 A ball is thrown with an initial velocity of 30 m/s at an angle of 25° above the horizontal, from a height of 1.5 m above the ground.

(a) Calculate how long the ball is in the air before it hits the ground.

(b) Find the length of time the ball is at least 5 m above the ground.

(c) Calculate the particle's horizontal range.

E.g. 3 A body is projected with velocity $(2\mathbf{i} + 11\mathbf{j})$ m/s from the ground. Find:

(a) the velocity of the body 0.8 s after projection

(b) the time of flight

(c) the maximum height.

Video: Finding maximum height and range
Video: Finding the range given the maximum height
Video: Formula for maximum height
Video: Range of projectile

Solutions to Starter and E.g.s

Exercise

p455 20A Qu 5-11

Additional questions

1. A ball is kicked with initial velocity 23 m/s, at an angle θ to the horizontal. The ball reaches the maximum height of 4.8 m. Find the value of θ.

2. An object is fired from a point 4 m above the ground. It has an initial speed of 20 m/s at an angle of 45° above the horizontal. For how long it is higher than 11 m above the ground.

3. A particle is projected from a point 0.6 m above the ground. Its initial speed is 7.5 m/s at an angle of α above the ground. Given that it reaches a maximum height of 2.8, find the horizontal distance travelled after 1.2 s.

4. A particle is fired from a point 16 m above the ground. Its initial speed is 3 m/s at an angle of 7^o below the horizontal. Find
   (a) the horizontal range of the particle
   (b) the speed and direction of the particle when it lands.

5. A cricket ball is hit when it is 0.5 m above ground. Its initial speed is 19 m/s, at an angle of 28^o. It is caught on its descent by a fielder when it is 2.5 m above the ground. Find:
   (a) the length of time it was in the air
   (b) the fielder's horizontal distance from where the ball was struck
   (c) the speed of the ball at the point where it was caught.

Summary

Maximum height, s_y: when the ***vertical velocity is zero*** i.e. $v_y = 0$.
Use $v^2 = u^2 + 2as$ vertically

Range, s_x: if projected ***from the ground***, this is when $s_y = 0$
if projected ***from a balcony/cliff***, this is when $s_y = -h$ (where h is the initial height)

1. Use $s = ut + \frac{1}{2}at^2$ (vertically), put it equal to zero and solve for t to find the time the object is in the air.
2. Use the $t-$value from step 1 and substitute it into $s = ut + \frac{1}{2}at^2$ (horizontally), remembering that $a_x = 0$.