Related Rates of Change

Starter

1. **(Review of last lesson)** A curve has parametric equations $x = 3t + 6$ and $y = 2t - 8$. Find the area between the curve, the $x-$axis and the points where $t = -2$ and $t = 2$.

Reminder: The chain rule is $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

2. Let $y = 3x^2$. The ***rate of change*** of y with respect to time, t, is 24. Use the ***chain rule*** to find the ***rate of change*** of x with respect to time when $x = 2$.

3. A circle with area A is expanding at a rate of 5 cm^2 per second (5 cm^2 s^{-1}). Find the exact rate of increase of the radius, r, of the circle when the radius is 4 cm.

Notes

Again we use the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$, when solving related rates of change problems.

Useful formulae

- Volume of sphere $= \dfrac{4}{3}\pi r^3$
- Surface area of sphere $= 4\pi r^2$
- Volume of a cone $= \dfrac{1}{3}\pi r^2 h$

N.B. It is important to include units in your answer

E.g. 1 Air is pumped into a spherical ball of volume V which expands at a rate of 8 cm^3 s^{-1}. Find the exact rate of increase of the radius, r, of the ball when the radius is 2 cm.

Working: "Find the exact rate of increase of the radius" means find $\dfrac{dr}{dt}$.

A spherical ball which expands at a rate of 8 cm^3 s^{-1} $\Rightarrow$ $\dfrac{dV}{dt} = 8$

By the chain rule: $\dfrac{dr}{dt} = \dfrac{dV}{dt} \times \dfrac{dr}{dV} = \dfrac{\frac{dV}{dt}}{\frac{dV}{dr}}$ ***so we need*** $\dfrac{dV}{dr}$

For a sphere $V = \dfrac{4}{3}\pi r^3$ $\Rightarrow$ $\dfrac{dV}{dr} = 4\pi r^2$

$\therefore \dfrac{dr}{dt} = \dfrac{\frac{dV}{dt}}{\frac{dV}{dr}} = \dfrac{8}{4\pi r^2} = \dfrac{2}{\pi r^2}$

So when $r = 2$, $\dfrac{dr}{dt} = \dfrac{2}{\pi \times 2^2} = \dfrac{1}{2\pi}$ cm s^{-1}

E.g. 2 A cuboid has dimensions 6, x and x. The volume of the cuboid, V, is increasing at a rate of 4 cm^3 s^{-1}. Find the rate of increase of the side x when its length is 11 cm.

Video: Related rates of change

Related rates of change EQ

Solutions to Starter and E.g.s

Exercise
p268 12E Qu 1i, 2i, 3i, 4-9

Summary

Use the chain rule, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$