

Related Rates of Change

Starter

1. **(Review of last lesson)** A curve has parametric equations $x = 3t + 6$ and $y = 2t - 8$. Find the area between the curve, the x -axis and the points where $t = -2$ and $t = 2$.

Reminder: The chain rule is $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

2. Let $y = 3x^2$. The **rate of change** of y with respect to time, t , is 24. Use the **chain rule** to find the **rate of change** of x with respect to time when $x = 2$.
3. A circle with area A is expanding at a rate of 5 cm^2 per second ($5 \text{ cm}^2 \text{ s}^{-1}$). Find the exact rate of increase of the radius, r , of the circle when the radius is 4 cm.

Notes

Again we use the chain rule, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, when solving related rates of change problems.

Useful formulae

- Volume of sphere = $\frac{4}{3}\pi r^3$
- Surface area of sphere = $4\pi r^2$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$

N.B. It is important to include units in your answer

E.g. 1 Air is pumped into a spherical ball of volume V which expands at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$. Find the exact rate of increase of the radius, r , of the ball when the radius is 2 cm.

Working: "Find the exact rate of increase of the radius" means find $\frac{dr}{dt}$.

A spherical ball which expands at a rate of $8 \text{ cm}^3 \text{ s}^{-1} \Rightarrow \frac{dV}{dt} = 8$

By the chain rule: $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}}$ **so we need $\frac{dV}{dr}$**

For a sphere $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$

$$\therefore \frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} = \frac{8}{4\pi r^2} = \frac{2}{\pi r^2}$$

So when $r = 2$, $\frac{dr}{dt} = \frac{2}{\pi \times 2^2} = \frac{1}{2\pi} \text{ cm s}^{-1}$

E.g. 2 A cuboid has dimensions 6, x and x . The volume of the cuboid, V , is increasing at a rate of $4 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the side x when its length is 11 cm.

E.g. 3 The volume of a pyramid, V , is increasing at a rate of $16 \text{ m}^3/\text{s}$. At this time, the area of the base, A , is 4 m^2 and it is increasing at a rate of $10 \text{ m}^2/\text{s}$. Given also that the height, h , is 2 m, calculate the rate of change of height.

N.B. Volume of a pyramid = $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$

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Exercise

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Summary

Use the chain rule, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dx} \cdot \frac{dx}{dt}$.

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$