

## Repeated integration by parts

### Starter

1. (Review of last lesson) Find  $\int_0^{\frac{\pi}{3}} 18x \sin 3x dx$ .

2. Find  $\int x^2 e^x dx$ .

### Notes

With some functions we need to do integration by parts twice – as with question 2 from the starter.

**E.g. 1** Find: (a)  $\int x^2 \sin x dx$       (b)  $\int_0^1 x^2 e^{-2x} dx$

**Working:** (a) Let  $u = x^2 \Rightarrow u' = 2x$   
Let  $v' = \sin x \Rightarrow v = -\cos x$

Using  $\int uv' = uv - \int u'v$ :

$$\begin{aligned}\int x^2 \sin x dx &= -x^2 \cos x - \int 2x \times (-\cos x) dx \\ &= -x^2 \cos x + \int 2x \cos x dx\end{aligned}$$

*Since  $\int 2x \cos x dx$  cannot be done directly, integration by parts is used again.*

Let  $u = 2x \Rightarrow u' = 2$   
Let  $v' = \cos x \Rightarrow v = \sin x$

Using  $\int uv' = uv - \int u'v$ :

$$\begin{aligned}\int x^2 \sin x dx &= -x^2 \cos x + \left( 2x \sin x - \int 2 \sin x dx \right) \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + c\end{aligned}$$

**E.g. 2\*** Find  $\int e^x \sin x dx$ .

**Video:** [Repeated integration by parts](#)

[Solutions to Starter and E.g.s](#)

### Exercise

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**Summary**

Integration by parts: 
$$\int u v' = uv - \int u' v$$

$v'$  is usually the more complicated function, except when  $\ln x$  is included.  
Sometimes required integration by parts repeated times.

