Resolving Forces

Starter

1. **(Review of last lesson)** A javelin is thrown from a height of 1.5 m above the ground, at an angle of 38^o to the horizontal. It reaches a maximum height of 12.5 m.
   (a) Calculate the javelin's initial velocity in m/s to 1 d.p.
   (b) Calculate the horizontal distance travelled by the javelin to the nearest metre.

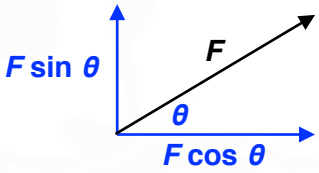
2. A force of 12 N acts at an angle of 30^o to the horizontal. How much of this force acts:
   (a) in the horizontal direction (b) in the vertical direction.

3. A force of R N acts at an angle of θ to the horizontal. How much of this force acts:
   (a) in the horizontal direction (b) in the vertical direction.

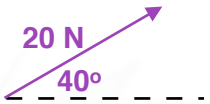
Notes

Resolving — sine or cosine

Turn through the angle ⇒ cos
Not turn through the angle ⇒ sin



E.g. 1 Find the horizontal and vertical components of the force shown.

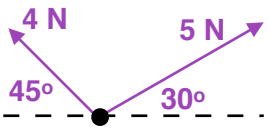


Resolving horizontally/vertically

- $R(\rightarrow) \equiv$ resolving horizontally — only looks at the forces acting horizontally
- $R(\uparrow) \equiv$ resolving vertically — only looks at the forces acting vertically

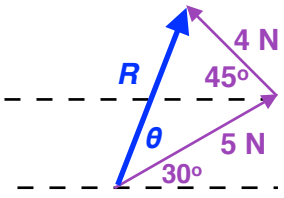
The resultant force of two forces can be found either by forming a triangle of forces or by resolving both in perpendicular directions (say, in the +ve $x-$ and $y-$directions). Once the resultant forces in these directions have been found, the overall resultant force can be found using Pythagoras.

E.g. 2 Find the magnitude and direction of the resultant force acting on the particle.



Working: ***Triangle of forces method***

The **thick blue line** is the resultant force, R.



Using the cosine rule:

$$R^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 75$$

$$R = 5.54$$

$$\frac{\sin\theta}{4} = \frac{\sin 75}{5.536} \quad \Rightarrow \quad \theta = 44.3^o$$

So the magnitude of the force is 5.54 N and the direction is $\theta = 74.3^o$

N.B. Remember, direction is measured in an anti-clockwise direction from the +ve $x-$axis.

Alternatively
Resolving in perpendicular directions method

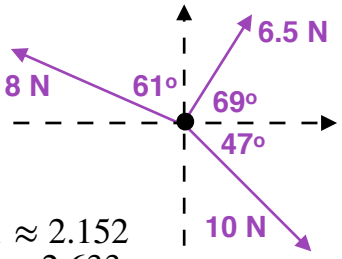
$R(\rightarrow)$: $\quad 5\cos 30 - 4\cos 45 \approx 1.502$
$R(\uparrow)$: $\quad 5\sin 30 + 4\sin 45 \approx 5.328$

By Pythagoras, the resultant, $R = \sqrt{1.502^2 + 5.328^2} = 5.54$ N

The angle is $\tan^{-1}\dfrac{5.328}{1.502} = 74.3^\circ$

When there are more than two forces acting, it is normal to resolve all forces in perpendicular directions.

E.g. 3 Find the magnitude and direction of the resultant force acting on the particle.



Working:

$R(\rightarrow)$: $\quad 6.5\cos 69 + 10\cos 47 - 8\sin 61 \approx 2.152$
$R(\uparrow)$: $\quad 8\cos 61 + 6.5\sin 69 - 10\sin 47 \approx 2.633$

By Pythagoras, the resultant, $R = \sqrt{2.152^2 + 2.633^2} = 3.40$ N (3 s.f.)

The angle is $\tan^{-1}\dfrac{2.633}{2.152} = 50.7^\circ$ (3 s.f.)

N.B. If an object is in equilibrium the resultant force in every direction must be zero.

E.g. 4 An object of mass 7 kg is held in equilibrium by two strings, A and B, attached to a ceiling. String A is at 40° to the horizontal and string B is at 50° to the horizontal. Find the tension in the two strings.

N.B. Objects in equilibrium (e.g. constant speed) — use $R(\rightarrow)$ and $R(\uparrow)$
Objects are accelerating — use $F = ma(\rightarrow)$ and $F = ma(\uparrow)$
If an object is in equilibrium the resultant force in every direction must be zero.

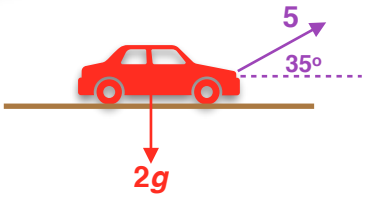
E.g. 5 A child pulls a toy car of mass 2 kg across a smooth floor with a force of 5 N at an angle of 35° to the horizontal. Find the acceleration of the car.

Working:

Smooth floor $\Rightarrow$ no friction
$F = ma(\rightarrow)$: $\quad 5\cos 35 = 2a$

$$a = \frac{5\cos 35}{2} = 2.04$$

The acceleration of the car is 2.04 m/s² (3 s.f.)



E.g. 6 A box of mass 15 kg is dragged along the floor at a constant speed of 1.2 m/s by means of a rope at 30° to the horizontal. The tension in the rope is 50 N. Calculate:
(a) the frictional force resisting the motion and
(b) the normal contact force from the floor.

Video: Resolving a force
Video: Resultant forces (2 forces at an angle)

Resultant forces EQ

Solutions to Starter and E.g.s

Exercise
p472 21A Qu 1i, 2ii, 3-8

Summary
Resolving forces: Turn through the angle ⇒ cos
Not turn through the angle ⇒ sin

$R(\rightarrow)$ ≡ resolving horizontally — only looks at the forces acting horizontally
$R(\uparrow)$ ≡ resolving vertically — only looks at the forces acting vertically

To find the resultant force of two or more forces resolving in perpendicular directions then use Pythagoras and trigonometry.

Objects in equilibrium (e.g. constant speed) — use $R(\rightarrow)$ and $R(\uparrow)$
Objects are accelerating — use $F = ma(\rightarrow)$ and $F = ma(\uparrow)$

If an object is in equilibrium the resultant force in every direction must be zero.