Review of factor theorem

Starter

1. (Review of previous material)
Factorise the polynomial $x^3 + 2x^2 - x - 2$ as far as possible.

Notes

From the AS course, the ***factor theorem*** states that:
"for a polynomial $f(x)$ if $f(a) = 0$ then $x - a$ is factor of $f(x)$".

This can be extended to: "if $f\left(\frac{b}{a}\right) = 0$ then $ax - b$ is factor of $f(x)$".

The factor theorem is a corollary of the ***remainder theorem*** which states that:
"when a polynomial $f(x)$ is divided by $x - a$, the remainder is $f(a)$"

In general: $f(x) \equiv (x - a) \times$ Quotient + Remainder

E.g. 1 Decide whether the linear expression is a factor of the polynomial $f(x)$:

(a) $x - 3$ $\quad f(x) = 4x^3 - 4x^2 - 21x - 10$
(b) $2x + 1$ $\quad f(x) = 2x^3 + 3x^2 + 3x + 1$

Working: (a) $f(3) = 4 \times 3^3 - 4 \times 3^2 - 21 \times 3 - 10 = -1 \neq 0$
Since $f(3) \neq 0$, $x - 3$ is not a factor.

E.g. 2 Factorise the polynomial $x^3 + x^2 - 16x + 20$ as far as possible.

Working: Let $f(x) = x^3 + x^2 - 16x + 20$
$f(2) = 2^3 + 2^2 - 16 \times 2 + 20 = 0$ so $x - 2$ is factor.
$x^3 + x^2 - 16x + 20 \equiv (x - 2)(ax^2 + bx + c)$
By inspection, $a = 1$ and $c = -10$
Equating coefficients of x^2: $\quad 1 = -2a + b$
Since $a = 1$, $b = 3$
N.B. a, b and c could have been found by polynomial division.
$x^3 + x^2 - 16x + 20 \equiv (x - 2)(x^2 + 3x - 10)$
Now factorise the quadratic:
$x^3 + x^2 - 16x + 20 \equiv (x - 2)(x - 2)(x + 5)$

E.g. 3 The polynomial $9x^3 + 27x^2 - x - 3$ is the product of three linear factors. Find them.

E.g. 4 The polynomial $x^3 + ax^2 + bx - 6$ has factors $x + 1$ and $x - 2$. Find the values of a and b.

E.g. 5 (a) Factorise $x^3 - 27$ into a linear and a quadratic factor.
(b) Hence or otherwise factorise: (i) $x^3 - y^3$ (ii) $x^3 + y^3$

Video: Factor Theorem

Exercise

p95 5A Qu 1i, 2-7, (8-10 red)

Summary

Factor Theorem: for a polynomial $f(x)$, if $f\left(\frac{b}{a}\right) = 0$ then $ax - b$ is factor of $f(x)$.

$x^3 - y^3 \equiv (x - y)(x^2 + xy + y^2)$

$x^3 + y^3 \equiv (x + y)(x^2 - xy + y^2)$