SUVAT in Vector Form

Starter

The equations of constant acceleration are:

$v = u + at$	No s
$s = \frac{1}{2}(u + v)t$	No a
$s = ut + \frac{1}{2}at^2$	No v
$s = vt - \frac{1}{2}at^2$	No u
$v^2 = u^2 + 2as$	No t

Remember: state what you know and choose the formula based on which letter is missing

1. **(Review of AS material)** A car is moving along a straight road with uniform acceleration. The car passes a check-point A with a speed of 12 m/s and another check-point C with a speed of 32 m/s. The distance between A and C is 1100 m.
   (a) Find the time taken, in seconds, taken by the car to move from A to C.
   (b) Given that B is the midpoint of AC, find, in m/s to 1 dp, the speed with which the car passes B.

Working:

(a) $u = 12, v = 32, s = 1100, t = ?$

No a $\Rightarrow$ $s = \frac{1}{2}(u + v)t$: $1100 = \frac{1}{2}(12 + 32)t$

$\therefore t = 50$ s

(b) From the journey from A to C, acceleration can be found:

$u = 12, v = 32, s = 1100, a = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $32^2 = 12^2 + 2a \times 1100$

$\therefore a = 0.4$

From A to B: $u = 12, s = 550, a = 0.4, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 12^2 + 2 \times 0.4 \times 550$

$v^2 = 584$

$\therefore v = 24.2$ m/s (3 s.f.)

2. **(Review of AS material)** A particle A starts from rest at a point O and moves on a straight line with constant acceleration 2m/s². At the same instant another particle B, 12 m behind O, is moving with velocity 5m/s and has a constant acceleration of 3 m/s². How far from O are the particles when B overtakes A?

Working:

Let S be the distance from O when B overtake A and the let the time be T.

Particle A: $u = 0, a = 2, t = T, s = S$

Particle B: $u = 5, a = 3, t = T, s = S + 12$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

Particle A: $S = \frac{1}{2} \times 2T^2$ $\Rightarrow$ $S = T^2$

Particle B: $S + 12 = 5T + \frac{1}{2} \times 3T^2$ $\Rightarrow$ $S + 12 = 5T + \frac{3}{2}T^2$

Replace S by T^2: $T^2 + 12 = 5T + \frac{3}{2}T^2$

Multiply by 2 and rearrange: $T^2 + 10T - 24 = 0$

$T = 2$ or $T = -12$ so $S = 4$

Distance from O is 4 m

Notes
The SUVAT equations can also be used in vector form.

In vector form the equations of constant acceleration (SUVAT) are:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \qquad \text{No } \mathbf{s}$$
$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t \qquad \text{No } \mathbf{a}$$
$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 \qquad \text{No } \mathbf{v}$$
$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 \qquad \text{No } \mathbf{u}$$

N.B. When writing the vectors, make sure there is a tilde (squiggle) under them.
Time, t, is not a vector and so is not in bold

Since **s** is now a vector, it is displacement and not distance. To find distance we would need to find the magnitude of the vector **s**.

$\mathbf{s}$ is the ***displacement*** vector ***from the starting point*** — it does not give current position, unless the object started from the origin

N.B. The ***velocity vector*** gives the ***direction of motion*** of the object.
The vector version of $v^2 = u^2 + 2as$ is not included because we do not learn how to multiply two vectors at A level maths — it appears only in A level further maths.

E.g. 1 A particle, P, has initial velocity $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ms^{-1} and constant acceleration $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ ms^{-2}.

(a) Find the displacement and distance from the starting position after 4 seconds.

(b) Given that P started from the point with position vector $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$, find its position after 4 seconds.

(c) Calculate the direction of motion after 5 seconds and calculate its speed at this time.

(d) Decide whether the the particle passes through the point $(0, 16)$.

N.B. A hint is given for each question

Working:

(a) Displacement will be the vector $\mathbf{s}$ and its magnitude will be the distance…

(b) To find its position, the displacement vector needs to be added to its starting position…

(c) The direction of motion is equal to the velocity at that moment.
The speed is the magnitude of the velocity….

(d) To pass through $(0, 16)$, initial vector + displacement $= \begin{pmatrix} 0 \\ 16 \end{pmatrix}$…

Video: SUVAT in 2-D
Video: SUVAT in 2-D Example

Solutions to Starter and E.g.s

Exercise
p432 19B Qu 1i, 2-8

Summary
SUVAT equations in 2-D:

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	No **s**
$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$	No **a**
$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	No **v**
$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$	No **u**