

SUVAT in Vector Form

Starter

The equations of constant acceleration are:

$$v = u + at \quad \text{No } s$$

$$s = \frac{1}{2}(u + v)t \quad \text{No } a$$

$$s = ut + \frac{1}{2}at^2 \quad \text{No } v$$

$$s = vt - \frac{1}{2}at^2 \quad \text{No } u$$

$$v^2 = u^2 + 2as \quad \text{No } t$$

Remember: state what you know and choose the formula based on which letter is missing

1. **(Review of AS material)** A car is moving along a straight road with uniform acceleration. The car passes a check-point A with a speed of 12 m/s and another check-point C with a speed of 32 m/s. The distance between A and C is 1100 m.
- (a) Find the time taken, in seconds, taken by the car to move from A to C.
- (b) Given that B is the midpoint of AC, find, in m/s to 1 dp, the speed with which the car passes B.

Working: (a) $u = 12, v = 32, s = 1100, t = ?$
 No $a \Rightarrow s = \frac{1}{2}(u + v)t: \quad 1100 = \frac{1}{2}(12 + 32)t$
 $\therefore t = 50 \text{ s}$

(b) From the journey from A to C, acceleration can be found:
 $u = 12, v = 32, s = 1100, a = ?$
 No $t \Rightarrow v^2 = u^2 + 2as: \quad 32^2 = 12^2 + 2a \times 1100$
 $\therefore a = 0.4$
 From A to B: $u = 12, s = 550, a = 0.4, v = ?$
 No $t \Rightarrow v^2 = u^2 + 2as: \quad v^2 = 12^2 + 2 \times 0.4 \times 550$
 $v^2 = 584$
 $\therefore v = 24.2 \text{ m/s (3 s.f.)}$

2. **(Review of AS material)** A particle A starts from rest at a point O and moves on a straight line with constant acceleration 2m/s^2 . At the same instant another particle B, 12 m behind O, is moving with velocity 5m/s and has a constant acceleration of 3m/s^2 . How far from O are the particles when B overtakes A?

Working: Let S be the distance from O when B overtakes A and let the time be T .

Particle A: $u = 0, a = 2, t = T, s = S$

Particle B: $u = 5, a = 3, t = T, s = S + 12$

No $v \Rightarrow s = ut + \frac{1}{2}at^2$

Particle A: $S = \frac{1}{2} \times 2T^2 \Rightarrow S = T^2$

Particle B: $S + 12 = 5T + \frac{1}{2} \times 3T^2 \Rightarrow S + 12 = 5T + \frac{3}{2}T^2$

Replace S by T^2 : $T^2 + 12 = 5T + \frac{3}{2}T^2$

Multiply by 2 and rearrange: $T^2 + 10T - 24 = 0$

$T = 2$ or $T = -12$ so $S = 4$

Distance from O is 4 m

Notes

The SUVAT equations can also be used in vector form.

In vector form the equations of constant acceleration (SUVAT) are:

$$\begin{array}{ll} \mathbf{v} = \mathbf{u} + \mathbf{a}t & \text{No } \mathbf{s} \\ \mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t & \text{No } \mathbf{a} \\ \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 & \text{No } \mathbf{v} \\ \mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 & \text{No } \mathbf{u} \end{array}$$

N.B. When writing the vectors, make sure there is a tilde (squiggle) under them.
Time, t , is not a vector and so is not in bold

Since \mathbf{s} is now a vector, it is displacement and not distance. To find distance we would need to find the magnitude of the vector \mathbf{s} .

\mathbf{s} is the **displacement** vector **from the starting point** — it does not give current position, unless the object started from the origin

N.B. The **direction of motion** is given by the **direction** of the **velocity vector**. i.e. the direction the velocity vector makes with the positive x -axis.
The vector version of $v^2 = u^2 + 2as$ is not included because we do not learn how to multiply two vectors at A level maths — it appears only in A level further maths.

E.g. 1 A particle, P , has initial velocity $\begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ms}^{-1}$ and constant acceleration $\begin{pmatrix} 1 \\ 5 \end{pmatrix} \text{ms}^{-2}$.

- Find the displacement and distance from the starting position after 4 seconds.
- Given that P started from the point with position vector $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$, find its position after 4 seconds.
- Calculate the direction of motion after 5 seconds and calculate its speed at this time.
- Decide whether the particle passes through the point $(0, 16)$.

N.B. A hint is given for each question

- Working:**
- Displacement will be the vector \mathbf{s} and its magnitude will be the distance...
 - To find its position, the displacement vector needs to be added to its starting position...
 - The direction of motion is given by the direction of the velocity vector. The speed is the magnitude of the velocity....
 - To pass through $(0, 16)$, initial vector + displacement = $\begin{pmatrix} 0 \\ 16 \end{pmatrix}$...

Video: [SUVAT in 2-D](#)
Video: [SUVAT in 2-D Example](#)

Exercise

p432 19B Qu 1i, 2-8

Summary

SUVAT equations in 2-D:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \quad \text{No } \mathbf{s}$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t \quad \text{No } \mathbf{a}$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 \quad \text{No } \mathbf{v}$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 \quad \text{No } \mathbf{u}$$