Set notation and Venn diagrams

Starter

Key probability formulae

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A') = 1 - P(A)$$

If A and B are ***mutually exclusive***: $P(A \cap B) = 0 \quad \Rightarrow \quad P(A \cup B) = P(A) + P(B)$

If A and B are ***independent***: $P(A \cap B) = P(A) \times P(B)$

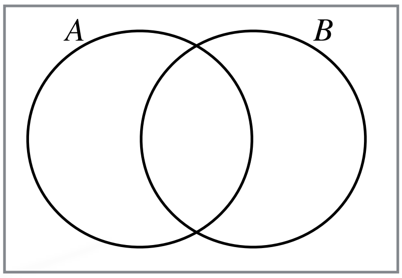
1. The events A and B are such that $P(A) = 0.4$, $P(B) = 0.45$ and $P(A \cup B) = 0.68$. Show that the events A and B are neither mutually exclusive nor independent.

2. *(Review of last lesson)* Draw three diagrams similar to the one on the right and shade the area given by:

   (a) $A \cap B$ (i.e. A intersect B)

   (b) $A \cup B$ (i.e. A union B)

   (c) A' (i.e. A complement)



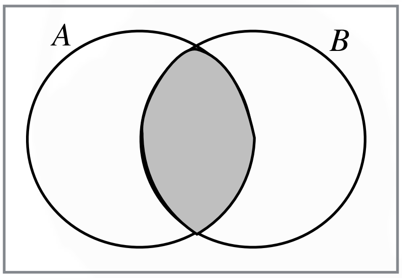
Notes

Key set notation

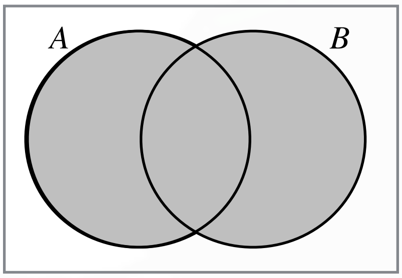
$\cap$ — ***intersection*** (or overlap, **AND**)

$\cup$ — ***union*** (or combine, **OR**)

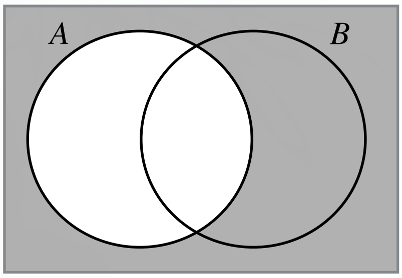
$'$ — ***complement***, **NOT** $A' \Rightarrow$ what is not in A



Intersect, $A \cap B$



Union, $A \cup B$



Complement, A'

This notation was met at GCSE.

Conditional probability

Conditional probability is the probability of an event when another event has already happened or when we restrict the number of objects under consideration.

"Given" or "if" usually introduce the part that has already happened and are denoted by $|$.
i.e. probability of B happening given that A has already happened $\equiv P(B|A)$

Conditional probability formula: $$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

E.g. 1 Two events X and Y are such that $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \cap Y) = 0.3$. Find:
(a) $P(X \cup Y)$ (b) $P(X \mid Y)$ (c) $P(Y \mid X)$

Working: (a) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
$= 0.7 + 0.4 - 0.3$
$= 0.8$

(b) $$P(X \mid Y) = \frac{P(Y \cap X)}{P(Y)} = \frac{0.3}{0.4} = \frac{3}{4}$$

(c) $$P(Y \mid X) = \frac{P(X \cap Y)}{P(X)} = \frac{0.3}{0.4 + 0.3} = \frac{3}{7}$$

E.g. 2 Two events A and B are such that $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.55$. Find:
(a) $P(A \mid B)$ (b) $P(B \mid A)$

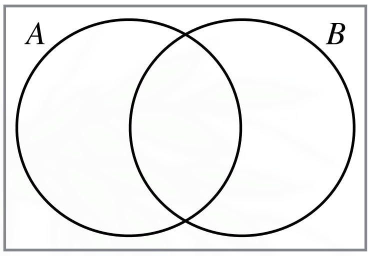
How would we calculate $P(A \mid B')$ for ***E.g. 2*** above?

In such cases it is best to draw a Venn diagram

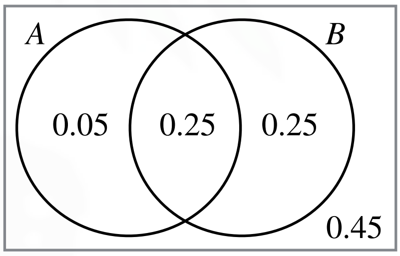
Conditional probability and Venn diagrams

With Venn diagrams, conditional probability reduces the number in the denominator from the total number of objects to only the part under consideration.

E.g. 3 Two events A and B are such that $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.55$.
(a) Copy and complete the Venn diagram.
(b) Hence calculate:
(i) $P(A \mid B')$
(ii) $P(B' \mid A')$



Working: (a) Since $P(A \cup B) = 0.55$, 0.45 goes on the outside.
$P(A \cap B) = 0.25$ so 0.25 goes in the intersection.
Number in A but not in the intersection is $0.3 - 0.25 = 0.05$.
Number in B but not in the intersection is $0.5 - 0.25 = 0.25$.



(b) (i) $$P(A \mid B') = \frac{P(B' \cap A)}{P(B')} = \frac{0.05}{0.05 + 0.45} = \frac{1}{10}$$

(ii) $$P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{0.45}{0.25 + 0.45} = \frac{9}{14}$$

E.g. 4 Two events X and Y are such that $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \cap Y) = 0.3$. Find:
(a) Draw a Venn diagram. (b) $P(X \mid Y')$ (c) $P(X' \mid Y)$

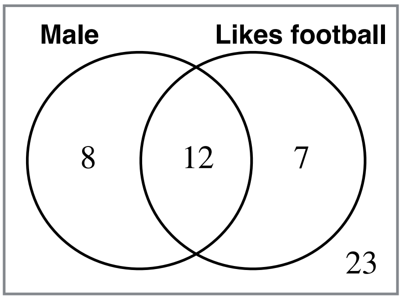
E.g. 5 A person is picked at random. Find the probability that:

(a) they are male given they like football

(b) they like football if they are female

(c) they don't like football given that they are female

(d) they are female given that they don't like football

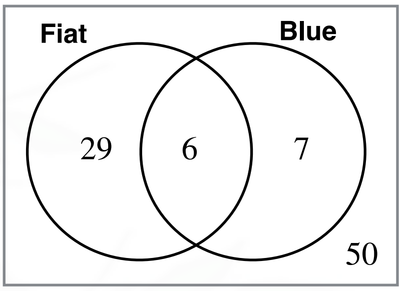


Working: (a) "given they like football" means we only consider this part of the diagram so that the denominator is $12 + 7$.
The numerator is either 12 or 7.
Since we want the males, it must be 12

$$P(\text{male} \mid \text{like football}) = \frac{12}{12+7} = \frac{12}{19}$$

E.g. 6 A car is picked at random. Find the probability that:

(a) it is a fiat given that it is blue.
(b) if is is not blue, then it is not a Fiat.
(c) it is not blue given it is a Fiat.



E.g. 7 At a sports club, members can play tennis and squash amongst other sport. $65\ \%$ of members play tennis, $28\ \%$ play squash and $31\ \%$ play neither.

(a) Calculate what percentage of members play both tennis and squash and draw a Venn diagram and.
(b) A member of the sports club is chosen at random. Find the probability that they:
  (i) play tennis or squash but not both.
  (ii) play squash given that they play tennis
  (iii) play tennis given that they don't play squash.

Video: Venn diagrams
Video: Three set problems
Video: Notation and defining regions
Video: Probability in Venn diagrams
Video: Conditional probability in Venn diagrams
Video: P(A∪B) and Mutually Exclusive Events

Exam questions: Venn diagrams

Solutions to Starter and E.g.s

Exercise
p360 16A Qu 1ace…, 2i, 3i, 4ace…, 5i, 6-19

Summary

$P(A') = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are ***mutually exclusive***: $P(A \cap B) = 0 \quad \Rightarrow \quad P(A \cup B) = P(A) + P(B)$

If A and B are ***independent***: $P(A \cap B) = P(A) \times P(B)$

$\cap$ — ***intersection*** (or overlap, **AND**)

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Conditional probability formula: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$