Sigma Notation

Starter

1. **(Review of last lesson)**

   A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 1 - \dfrac{1}{u_n}$ for $n \geq 1$.

   (a) White down the values of u_2, u_3 and u_4.

   (b) Describe the behaviour of the sequence.

2. **(Review of last lesson)** A farmer decides to plant a number of tree as a boundary to a field. The trees are expected to grow by 75 cm per year, so he cuts 20% off their height at the start of each year.

   (a) Express this as a linear recurrence relation.

   (b) What is the maximum height the trees should grow to?

   (c) What percentage of tree height should be cut off each year if he wishes the tree to not exceed 2.5 m?

3. A sequence has nth term is $an^2 + b$, where a and b are constants.

   (a) If the 3rd term is 7 and the 5th term is 23, find the values of a and b.

   (b) Is 35 in the sequence?

Notes

Sequence vs. series

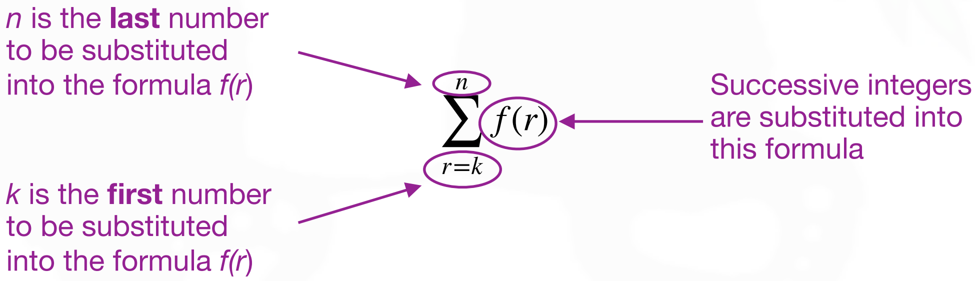
Sequence: $u_1, u_2, u_3, \ldots, u_n$

Series: $u_1 + u_2 + u_3 + \ldots + u_n$

Generating a sequence from sigma notation

The capital letter Σ ("Sigma") from the Greek alphabet is used to denote "the sum of" and is a particularly neat way of expressing a series.

Consecutive integers are substituted into the formula $f(r)$ starting with $r = k$ and finishing with $r = n$.



$$\sum_{r=k}^{n} f(r) = f(k) + f(k+1) + f(k+2) + \ldots + f(n-1) + f(n)$$

Number of terms in a sequence $= n - k + 1$

E.g. 1 Write out the individual terms of the series and hence evaluate $\sum_{r=3}^{7} (2r - 1)$.

Working:

$$\sum_{r=3}^{7} (2r-1) = (2\times3-1) + (2\times4-1) + (2\times5-1) + (2\times5-1) + (2\times7-1)$$
$$= 5 + 7 + 9 + 11 + 13 = 45$$

E.g. 2 By writing the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots+\frac{1}{50}$ in at least 2 sigma notation ways show that there is not a unique way of expressing a series in sigma notation.

Dealing with alternate signs

How can we express the series 1 − 2 + 3 − 4 + 5 in sigma notation?

1st consider it without the negative signs. Then either $(-1)^r$ or $(-1)^{r+1}$.

$(-1)^r = - + - + \ldots$

$(-1)^{r+1} = + - + - \ldots$

E.g. $1-2+3-4+5=\sum_{r=1}^{5}(-1)^{r+1}r$

E.g. 3 Write out the individual terms of these series and hence find their value:

(a) $\sum_{r=0}^{5} r(r+1)$ (b) $\sum_{r=1}^{4}(-1)^r(r+3)$

Working: (a) $\sum_{r=0}^{5} r(r+1) = 0+1(1+1)+2(2+1)+3(3+1)+4(4+1)+5(5+1)$

$= 0+2+6+12+20+30=70$

E.g. 4 Express these series in sigma notation

(a) 28 + 22 + 16 + 10 + …. (b) 2 − 4 + 6 − … − 20

Working: (a) Term-to-term rule is −6 $\Rightarrow$ $-6r$

Term before the first term is $28+6=34$ $\Rightarrow$ $-6r+34$

$28+22+16+10+\ldots. = \sum_{r=1}^{\infty}(34-6r)$

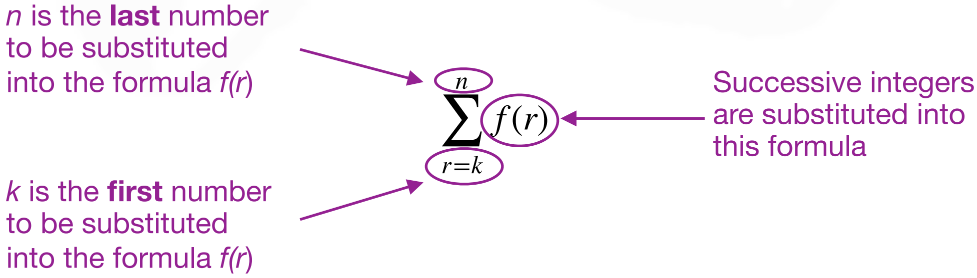
Video: Sigma notation

Solutions to Starter and E.g.s

Exercise

p69 4B Qu 1i, 2i, 3-6

Summary



$$\sum_{r=k}^{n} f(r) = f(k)+f(k+1)+f(k+2)+\ldots+f(n-1)+f(n)$$

Number of terms in a sequence $= n-k+1$