Simplifying rational expression

Starter

1. **(Review of last lesson)** The polynomial $px^3 + 5x^2 + qx + 8$ has factors $3x - 1$ and $x + 4$. Find the values of p and q.

2. **(Review of GCSE material)**

Simplify: (a) $\dfrac{x^2 + 5x - 6}{x^2 - 4x + 3}$ (b) $\dfrac{x^2 - 4}{x + 3} \times \dfrac{3}{x - 2}$

Notes

When simplifying rational functions (called algebraic fractions at GCSE), ***factorise*** where possible and ***then cancel***.

Rational functions involving multiplication and division are similar to numbers.

Multiplication — multiply the numerators, multiply the denominators

Division — find the reciprocal of the second fraction (i.e. flip it) and multiply the fractions.

E.g. 1 Simplify: (a) $\dfrac{x + 2}{2x + 3} \div \dfrac{2x + 4}{8x + 12}$ (b) $\dfrac{5x - 1}{2x^2 + x - 3} \div \dfrac{1}{2x^2 + 7x + 6}$

Working: (a) $$\frac{x + 2}{2x + 3} \div \frac{2x + 4}{8x + 12} = \frac{x + 2}{2x + 3} \times \frac{8x + 12}{2x + 4}$$
$$= \frac{x + 2}{2x + 3} \times \frac{4(2x + 3)}{2(x + 2)}$$
$$= \frac{4}{2} = 2$$

Alternative to polynomial division

When $x^2 - 5x + 6$ is divided by $x + 1$, the ***quotient*** is $x - 6$ and the ***remainder*** is 12. In this example, $x^2 - 5x + 6$ is called the ***dividend*** and $x + 1$ is the ***divisor***.

Here is a comparison of numbers, words and algebra:

Numbers	Words	Algebra
$\dfrac{23}{5} = 4 + \dfrac{3}{5}$	$\dfrac{\text{Dividend}}{\text{Divisor}} \equiv \text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}}$	$\dfrac{x^2 - 5x + 6}{x + 1} \equiv x - 6 + \dfrac{12}{x + 1}$
Multiply by 5:	Multiply by the divisor:	Multiply by $x + 1$:
$23 = 5 \times 4 + 3$	Dividend $\equiv$ Divisor $\times$ Quotient $+$ Remainder	$x^2 - 5x + 6 \equiv (x + 1)(x - 6) + 12$

Since $x^2 - 5x + 6$ is of degree 2 and $x - 6$ is of degree 1, we know that the quotient will be of degree 1 and of the form $Ax + B$ and the remainder will be a constant term, say C.

i.e. $$\frac{x^2 - 5x + 6}{x + 1} \equiv Ax + B + \frac{C}{x + 1}$$

Multiply by $x + 1$: $$x^2 - 5x + 6 \equiv (x + 1)(Ax + B) + C$$

By equating coefficients, the values of A, B and C can be found.

x^2: $1 = A$

x: $-5 = A + B$ Since $A = 1$, $B = -6$

constant: $6 = B + C$ Since $B = -6$, $C = 12$

So $\dfrac{x^2 - 5x + 6}{x + 1} \equiv x - 6 + \dfrac{12}{x + 1}$ and the quotient is $x - 6$ and the remainder is 12

N.B. **Degree of quotient = Degree of dividend — Degree of divisor**
Degree of remainder = Degree of divisor — 1

E.g. 2 Without using polynomial division, find the quotient and remainder when:

(a) $2x^2 + 9x - 4$ is divided by $x - 2$

(b) $x^3 + 4x^2 - 7$ is divided by $x^2 - 3$

(c) $3x^3 - 5$ is divided by $x + 4$

Working: (a) $\dfrac{2x^2 + 9x - 4}{x - 2} \equiv Ax + B + \dfrac{C}{x + 1}$

$2x^2 + 9x - 4 \equiv (x - 2)(Ax + B) + C$

Equating coefficient: x^2: $2 = A$

x: $9 = -2A + B$ $\therefore B = 13$

constant: $-4 = -2B + C$ $\therefore C = 22$

$\dfrac{2x^2 + 9x - 4}{x - 2} \equiv 2x + 13 + \dfrac{22}{x + 1}$

The quotient is $2x + 13$ and the remainder is 22.

Video: Simplifying algebraic fractions
Video: Multiplication of algebraic fractions

Simplifying algebraic fractions EQ
Algebraic long division EQ

Solutions to Starter and E.g.s

Exercise
p97 5B Qu 1i, 2i, 3i, 4i, 5-13, (14-16 red)

Summary
Rational functions involving multiplication and division are similar to numbers.

Multiplication — multiply the numerators, multiply the denominators

Division — find the reciprocal of the second fraction (i.e. flip it) and multiply the fractions.

$\dfrac{\text{Dividend}}{\text{Divisor}} \equiv \text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}} \Rightarrow \text{Dividend} \equiv \text{Divisor} \times \text{Quotient} + \text{Remainder}$

Usually: Degree of quotient = Degree of dividend — Degree of divisor
Degree of remainder = Degree of divisor — 1