Solving Geometrical Problems

Starter

1. **(Review of last lesson)** Decide, with clear working, whether the points $A(1, 0, 3)$, $B(3, 1, 2)$ and $C(7, 3, 0)$ are collinear.

2. **(Review of last lesson)** P is the point $(2, -1, 4)$ and Q is the point $(q-2, 5, 2q+1)$. Given that the length of PQ is 11, find the possible coordinates of Q.

Notes

Reminder from AS:

Unit vectors, $\hat{\mathbf{v}}$, have a length of 1 unit: $\mathbf{v} = |\mathbf{v}| \times \hat{\mathbf{v}} \quad \Rightarrow \quad \hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

Midpoint: the midpoint of the line segment AB is $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

Parallel vectors: are ***multiples*** of one another i.e. $\mathbf{a}$ and $\mathbf{b}$ are parallel if $\mathbf{a} = k\mathbf{b}$

E.g. 1 Find the unit vector in the direction of $4\mathbf{i} - 4\mathbf{j} - 7\mathbf{k}$.

E.g. 2 Find a vector of magnitude 12 units that is parallel to $-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.
Hint: find the unit vector then…

E.g. 3 Given that $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $\mathbf{c} = -6\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, find the value of λ such that $\lambda\mathbf{a} + \mathbf{b}$ is parallel to $\mathbf{c}$.

E.g. 4 M is the midpoint of the line CD, where C $(-1, 3, -5)$ and $\overrightarrow{CD} = \begin{pmatrix} 4 \\ -4 \\ 6 \end{pmatrix}$. Find the coordinates of M.

Video A: Vector geometry
Video B: Vector geometry

Solutions to Starter and E.g.s

Exercise

p446 19E Qu 1i, 2-10

Summary

Unit vector, $\hat{\mathbf{v}}$: $\mathbf{v} = |\mathbf{v}| \times \hat{\mathbf{v}} \quad \Rightarrow \quad \hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

Midpoint: the midpoint of the line segment AB is $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

Parallel vectors are ***multiples*** of one another i.e. $\mathbf{a}$ and $\mathbf{b}$ are parallel if $\mathbf{a} = k\mathbf{b}$