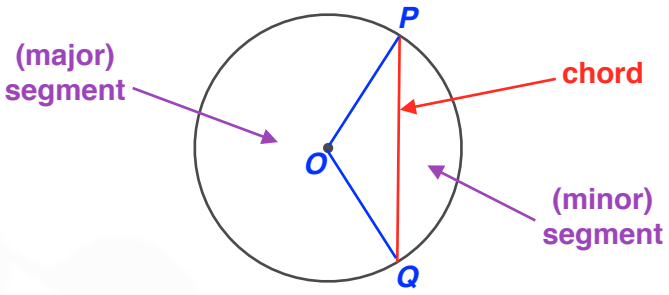
Triangles and circles

Starter

Area of the segment

A ***sector*** is the area between ***two radii and the circumference*** of the circle.

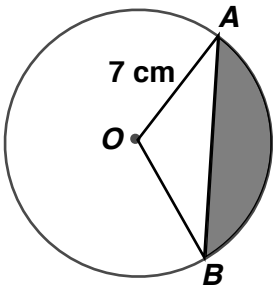
A ***segment*** is the area between ***a chord and the circumference*** of the circle.



1. Write down the formula for the area of a segment when the angle is given in radians.

2. Find the value of the shaded area given that $\angle AOB = \frac{2\pi}{3}$.

   Give your answer exactly.



Notes

Area of segment = Area of Sector − Area of Triangle

$$= \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin\theta^c$$

$$= \frac{1}{2}r^2\left(\theta^c - \sin\theta^c\right)$$

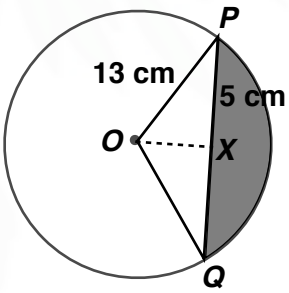
E.g. 1 Find the area of the segment cut off by a chord of length 10 cm from a circle of radius 13 cm.

Working: ***By drawing a dotted line, two right-angled triangles are created.***

$\sin P\hat{O}X = \frac{5}{13} \quad \Rightarrow \quad P\hat{O}X \approx 0.395^c$

So $P\hat{O}Q \approx 2 \times 0.395^c \approx 0.7896^c$

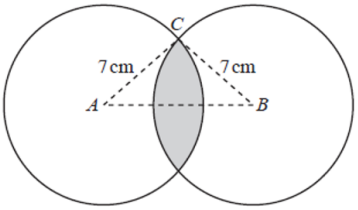
…



E.g. 2 The diagram shows two circles of radius 7 cm with centres A and B. The distance AB is 12 cm and the point C lies on both circles. The region common to both circles is shaded.

(a) Find the angle CAB in radians to 4 s.f..

(b) Find the area of the shaded region to 3 s.f..



Video: Arcs, sectors and segments

Arcs, sectors and segments EQ

Solutions to Starter and E.g.s

Exercise
p154 7E Qu 1-4, (5 red)

Summary
Area of segment = Area of Sector − Area of Triangle

$$= \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2 \sin\theta^c$$
$$= \frac{1}{2}r^2\left(\theta^c - \sin\theta^c\right)$$