

Vectors in 3-D

Starter

- (Review of last lesson)** A boat is modelled as a particle moving across a horizontal surface with acceleration $\mathbf{a} = (e^{0.2t}\mathbf{i} + \sin t\mathbf{j})$ m/s² at time t seconds for $0 \leq t \leq 5$. At time $t = 0$, the boat is at $2\mathbf{j}$ m from the origin, moving in the \mathbf{j} direction at a speed of 10 m/s. Find an expression for the boat's position vector, \mathbf{s} , at time t .
- (Review of AS material)** Let $\vec{OA} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} -7 \\ -8 \end{pmatrix}$. Find the vector \vec{AB} and hence find the distance between A and B.

Notes

Rules that apply to 2-D also work for 3-D. For example, the vector from A to B is $\mathbf{b} - \mathbf{a}$

$$\text{i.e. } \vec{AB} = \mathbf{b} - \mathbf{a}$$

E.g. 1 The triangle JKL has vertices at points J(4, 0, -3), K(-1, 3, 0) and L(2, 2, 7). Find the vectors \vec{JK} , \vec{KL} and \vec{LJ} .

Magnitude of a vector

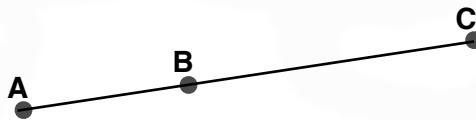
$$\text{If } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

E.g. 2 Given that $\mathbf{a} = \mathbf{i} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$, find the values of $\mu \in \mathbb{R}$ such that $\mathbf{a} - \mu\mathbf{b}$ has magnitude $3\sqrt{2}$.

Distance between 2 points

The distance between points with position vectors \mathbf{a} and \mathbf{b} is $|\mathbf{b} - \mathbf{a}|$

Success criteria – proving the collinearity of 3 points



- Find two different vectors connecting two pairs of points e.g. \vec{AB} and \vec{BC}
- Show that the two vectors are parallel by showing they are multiples of each other
- State that the vectors have a common point.

E.g. 3 Show that the points A, B and C are collinear if $\vec{OA} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\vec{OB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\vec{OC} = -\mathbf{i} + 8\mathbf{j} + 7\mathbf{k}$.

Video A: [Vectors in 3-D](#)
Video B: [Vectors in 3-D](#)

Exercise

p442 19D Qu 1i, 2i, 3i, 4ac, 5i, 6ac, 7ac, 8-12

Summary

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

Magnitude of a vector: if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \Rightarrow |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

The distance between points with position vectors \mathbf{a} and \mathbf{b} is $|\mathbf{b} - \mathbf{a}|$

Proving the collinearity of 3 points:

1. Find two different vectors connecting two pairs of points e.g. \overrightarrow{AB} and \overrightarrow{BC}
2. Show that the two vectors are parallel by showing they are multiples of each other
3. State that the vectors have a common point.