U6 Ma Mock Teacher X 19-20 SOLUTIONS [69]

1.

5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\}\ f'(x) = 6x^2 + 2x$	**B1**	**1.1b**
	$\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	**M1**	**1.1b**
	$= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$ *	**A1***	**2.1**
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}\ x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	**M1**	**1.1b**
	$\Rightarrow x_2 = \frac{3}{4},\ x_3 = \frac{2}{3}$	**A1**	**1.1b**
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	**B1**	**2.3**
		(1)	
			(6 marks)

2.

7 LHS is $k(x+y)(1+\frac{dy}{dx})$	**M1**	or $2x + 2y\frac{dy}{dx} + ky + kx\frac{dy}{dx}$ k is any positive integer	some terms may appear on RHS with signs reversed
$k = 2$	**A1**		if **M0** in middle scheme, **SC1** for **three terms out of four** completely correct with $k = 2$
$2y\frac{dy}{dx}$ on RHS from differentiating y^2	**B1**		may appear on LHS with sign reversed
$y^2 + Kxy\frac{dy}{dx}$ on RHS	**M1**	K is any positive integer	NB $K = 2$; may appear on LHS with signs reversed
obtains a value of y from eg $(1+y)^2 = 1 \times y^2$ oe	**M1**	allow even if follows incorrect manipulation	NB $y = -0.5$
substitution of $x = 1$ and their y dependent on at least two correct terms seen following differentiation, even if follows subsequent incorrect manipulation	**M1**	may be implied by $1 + \frac{dy}{dx} = \frac{1}{4} - \frac{dy}{dx}$	or $\frac{dy}{dx} = \frac{2-1-0.25}{-1-2+1}$ NB $\frac{dy}{dx} = \frac{2x+2y-y^2}{2xy-2x-2y}$
$\frac{dy}{dx} = -\frac{3}{8}$ oe cao	**A1** **[7]**		-0.375

3.

(a)	Uses $n\log_a x = \log_a x^n$ correctly	AO1.1a	M1	$\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$ $\Rightarrow \log_a y = \log_a 7^2 + \log_a 4 + \frac{1}{2}$ $= \log_a (49\times 4) + \frac{1}{2}$ $= \log_a 196 + \frac{1}{2}\log_a a$ $= \log_a 196 + \log_a \sqrt{a}$ $= \log_a 196\sqrt{a}$ $\therefore y = 196\sqrt{a}$
	Uses $\log_a x + \log_a y = \log_a xy$ or $\log_a x - \log_a y = \log_a \frac{x}{y}$ correctly	AO1.1a	M1	
	Obtains $\sqrt{a}$	AO1.1b	B1	
	Obtains correct answer in any correct form.	AO1.1b	A1	
(b)	Explains that $-\frac{3}{2}$ should be rejected as it is not possible to evaluate $\log_a\left(-\frac{3}{2}\right)$	AO2.3	E1	$-\frac{3}{2}$ should be rejected as it is not possible to evaluate $\log_a\left(-\frac{3}{2}\right)$
	Total		**5**	

4.

Models the rate of change of volume with a differential equation of the correct form. With respect to time, not contradicted.	3.3	B1	$\frac{dv}{dt} = k$
Obtains $4\pi r^2$ by differentiation.	1.1b	B1	$\frac{dv}{dr} = 4\pi r^2$
Uses the chain rule to connect rates of change substituting their expressions for dv/dt and dv/dr. Or Integrates to obtain expression for v=kt+c Then differentiates wrt r dv/dr=kdt/dr Substitutes their expression for dv/dr	3.1b	M1	$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$
Completes argument, obtaining a correct expression for $\frac{dr}{dt}$ and concluding that $\frac{dr}{dt} \propto \frac{1}{r^2}$ OE statement	2.1	R1	$k = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{k}{4\pi r^2}$ $\therefore \frac{dr}{dt} \propto \frac{1}{r^2}$
Total		**4**	

5.

7	(i)	$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of $x = 2\sin t$
		$\frac{dy}{dx} = \text{their}\ \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
		$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
		$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to $1 - 2\sin t$ www	A1	or equivalent intermediate step	
		(1, 1½)	B1 [5]	NB $t = \frac{\pi}{6}$	from $1 - 2\sin t = 0$
7	(ii)	(y =) $1 - 2\sin^2 t + 2\sin t$	B1	may be awarded after correct substitution for x eg (y =) $1 - {}^{x}/_{4} - \sin^2 t + 2\sin t$	or (y =) $x + \cos 2t$
		substitution of $\sin t$ = ½x to eliminate t	M1		substitution of $t = \sin^{-1}({}^{x}/_{2})$ to eliminate t
		$y = 1 + x - ½x^2$ oe isw	A1 [3]	or B3 www	$y = x + \cos 2(\sin^{-1}({}^{x}/_{2}))$ oe isw

6.

Integrates using integration by parts	AO3.1a	M1	$y = \int (x-1)e^x dx$ $u = x - 1 \quad \frac{dv}{dx} = 1$ $\frac{dv}{dx} = e^x \quad v = e^x$
Applies integration by parts formula correctly to either of $(x-1)e^x$ or xe^x	AO1.1a	M1	
Obtains fully correct integral, condone missing constant.	AO1.1b	A1	$y = (x-1)e^x - \int e^x dx$ $y = (x-1)e^x - e^x + c$
Explains clearly why the minimum y value is e with reference to the range of the function OE	AO2.4	E1	Range $\geq e \Rightarrow$ at min $y = e$
Uses $\frac{dy}{dx} = 0$ to find x coordinate of minimum	AO1.1a	M1	Min point when $\frac{dy}{dx} = 0 \therefore x = 1$
Deduces that the curve passes through the point $(1, e)$	AO2.2a	A1	So curve passes through $(1, e)$ $e = (1-1)e^1 - e^1 + c$
Uses their minimum point to find their c	AO1.1a	M1	$c = 2e$
States the correct equation in any correct form Condone y instead of $f(x)$ CAO	AO1.1b	A1	$\therefore f(x) = (x-2)e^x + 2e$
Total		8	

7.

Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution $u = \ldots$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
Substitution $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u^2\pm2\right)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u\pm2\right)\sqrt{u}\,\mathrm{d}u$	M1	1.1b
$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
		(7)	
			(7 marks)

Or

Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • using by parts the correct way around • and using limits	M1	3.1a
$\int\left(\sqrt{x+2}\right)dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
$\int 2x\sqrt{x+2}\,dx = Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$	M1	1.1b
$= Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$	dM1	2.1
$= \frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
Uses limits 2 and 0 the correct way around	ddM1	1.1b
$= \frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
	(7)	

8.

(a)	For a correct equation in p or q $\quad p = 10^{4.8}$ or $q = 10^{0.05}$	M1	1.1b
	For $p =$ awrt 63100 or $q =$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p =$ awrt 63100 and $q =$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$=$ awrt (£)2000000	A1	1.1b
		(2)	
			(8 marks)

9.

10(a)	$\frac{dH}{dt} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\,dH = \int \frac{\cos 0.25t}{40}\,dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t\,(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t = 0, H = 5 \Rightarrow c = \ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

10.

8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5$ m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$=(5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
			(8 marks)