U6 Ma Mock Teacher X 21-22 SOLUTIONS [74]

1.

$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
$\log_3\frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3(3^2(1-3y)) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
$y=\frac{4}{39}$	A1	1.1b
	(3)	

2.

(a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$	A1
		(4)
(b)	For $x<-1$ Follow through on their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x+1)^n}, n=1,3$	B1ft
		(1)

3.

Uses integration by parts with $u = x$ and $v' = \cos x$ **PI** by $x\sin x + \cos x$	3.1a	B1	$u = x \qquad u' = 1$ $v' = \cos x \qquad v = \sin x$ $\int x\cos x\,dx = x\sin x - \int \sin x\,dx$ $= x\sin x + \cos x$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x\,dx = [x\sin x + \cos x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{\pi}{3}\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - \left(\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$ $= \pi\frac{\sqrt{3}}{6} + \frac{1}{2} - \left(\pi\frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right)$ $= \left(\frac{4\sqrt{3} - 3\sqrt{2}}{24}\right)\pi + \left(\frac{1-\sqrt{2}}{2}\right)$
Applies integration by parts formula correctly by substituting their u, u', v and v' **PI** by $x\sin x + \cos x$	1.1a	M1	
Obtains $x\sin x + \cos x$ **CAO**	1.1b	A1	
Substitutes limits correctly into their integrated expression **PI** by correct a and b	1.1a	M1	
Uses correct exact value for any one of $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ or $\cos\frac{\pi}{3} = \frac{1}{2}$ or $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ or $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ **PI** by correct a or b	1.2	B1	
Obtains correct exact values of a and b **ACF** Ignore if 0.14(…) seen subsequently	1.1b	A1	
Total		**6**	

4.

$x^2 + 2xy + 2y^2 = 10$

$2x + 2y + 2x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$

Highest and lowest points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0 \Rightarrow x = -y$

$y^2 - 2y^2 + 2y^2 = 10$

$y = \pm\sqrt{10}$

$\therefore$Height $= \sqrt{10} - (-\sqrt{10})$

$= 2\sqrt{10} = 6.32$ m

M1	Attempts implicit differentiation; 3+ terms correct
A1	Completely correct; no need to get $\frac{dy}{dx} = \frac{-x-y}{x+2y}$
R1	States or implies that $\frac{dy}{dx} = 0$
A1	Finds $x = -y$
M1	Substitutes into the original equation
A1	Solves to get two y −values; allow FT
A1	Finds distance between their two y −values; allow FT

5.

(a)		Considers both $f(0.5)$ and $f(0.6)$ where $f(x)=\pm\{6\arcsin(2x-1)-x^2\}$	**M1**	**1.1**	With at least one correct value – values should be given to at least 2 sf (rot)	Allow degrees for **M1** only: $f(0.6)=68.8617...$
		$f(0.5)=-0.25<0,\ f(0.6)=0.8481...>0$ change of sign indicates that the root lies between 0.5 and 0.6	**A1**	**2.4**	Correct values together with explanation in words (change of sign) and conclusion	
			[2]			
(b)		$6\arcsin(2x-1)-x^2=0\Rightarrow\arcsin(2x-1)=\frac{1}{6}x^2$ So $2x-1=\sin\left(\frac{1}{6}x^2\right)$	**M1**	**1.1**	Correct order of operations to get $2x-1=\sin\left(kx^2\right)$	$k\neq 0$
		$x=\frac{1}{2}+\frac{1}{2}\sin\left(\frac{1}{6}x^2\right)$	**A1**	**2.2a**	$p=\frac{1}{2},\ q=\frac{1}{2}$ and $r=\frac{1}{6}$ (oe)	
			[2]			
(c)		$(x_0=0.5)$ $(x_1=)0.5208273057...$ $(x_2=)0.5225973903...$ $(x_3=)0.5227511445....$ $(x_4=)0.5227645245...$	**M1**	**1.1**	Uses their iterative formula with correct starting value to produce terms up to at least x_2 to at least 4 significant figures	Allow degrees for **M1** only: For reference: $x_1=0.5003636...$ $x_2=0.5003641...$ $x_3=0.5003641...$
		0.5228	**A1**	**1.1**	Must be stated to exactly **4** significant figures	
			[2]			

6.

10(a)	$T=al^b\Rightarrow\log_{10}T=\log_{10}a+\log_{10}l^b$	M1	2.1
	$\Rightarrow\log_{10}T=\log_{10}a+b\log_{10}l$ * or $\Rightarrow\log_{10}T=b\log_{10}l+\log_{10}a$ *	A1*	1.1b
		(2)	
(b)	$b=0.495$ or $b=\frac{45}{91}$	B1	2.2a
	$0=\text{"}0.495\text{"}\times-0.7+\log_{10}a\Rightarrow a=10^{0.346...}$ or $0.45=\text{"}0.495\text{"}\times 0.21+\log_{10}a\Rightarrow a=10^{0.346...}$	M1	3.1a
	$T=2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	

7.

12(a)	$$u=1+\sqrt{x}\Rightarrow x=(u-1)^2\Rightarrow\frac{dx}{du}=2(u-1)$$ or $$u=1+\sqrt{x}\Rightarrow\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$$	B1
	$$\int\frac{x}{1+\sqrt{x}}dx=\int\frac{(u-1)^2}{u}2(u-1)du$$ or $$\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times 2x^{\frac{1}{2}}du=\int\frac{2x^{\frac{3}{2}}}{u}du=\int\frac{2(u-1)^3}{u}du$$	M1
	$$\int_0^{16}\frac{x}{1+\sqrt{x}}dx=\int_1^5\frac{2(u-1)^3}{u}du$$	A1
		(3)
(b)	$$2\int\frac{u^3-3u^2+3u-1}{u}du=2\int\left(u^2-3u+3-\frac{1}{u}\right)du=\ldots$$	M1
	$$=(2)\left[\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right]$$	A1
	$$=2\left[\frac{5^3}{3}-\frac{3(5)^2}{2}+3(5)-\ln 5-\left(\frac{1}{3}-\frac{3}{2}+3-\ln 1\right)\right]$$	dM1
	$$=\frac{104}{3}-2\ln 5$$	A1
		(4)

8.

3(a)	$y = \text{cosec}^3\theta \Rightarrow \frac{dy}{d\theta} = -3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1
	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	$\frac{dy}{dx} = \frac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1
		(3)
(b)	$y = 8 \Rightarrow \text{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \frac{1}{8} \Rightarrow \sin\theta = \frac{1}{2}$	M1
	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \frac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{\frac{-3}{\sin^3\theta} \times \frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \frac{-3\times 8\times \frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1
	$= -24\sqrt{3}$	A1
		(3)

9.

7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times5^2 - 20\times5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, –23) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right)\,dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	

10.

4 (a)	Uses the model to state $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ (for positive constant c)	B1	3.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ and $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	M1	2.1
	$-c=4\pi r^2\times\frac{\mathrm{d}r}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{c}{4\pi r^2}=-\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{k}{r^2}\Rightarrow\int r^2\mathrm{d}r=\int -k\mathrm{d}t$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3}=-kt(+\alpha)$	A1	1.1b
	Uses $t=0, r=40\Rightarrow\alpha=\ldots$ $\alpha=\frac{64000}{3}$	M1	1.1b
	Uses $t=5, r=20$ & $\alpha=\ldots\Rightarrow k=\ldots$	M1	3.4
	$r^3=64000-11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. "$64000-11200t$" $\ldots 0\Rightarrow t\ldots$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	

11.

(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1
	$(6\mathbf{i} - 8\mathbf{j})$ (m s^{-1})	A1
		(2)
(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = \mathbf{0}$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ ($\mathbf{j}$ terms only)	A1
	The first two marks could be implied if they go straight to an algebraic equation.	
	Attempt to equate $\mathbf{j}$ components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1
	$T = 1.8$	A1
		(4)
(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the $\mathbf{i}$ component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1
		(2)