Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 1 (X)
February 2025
2024-2025

Duration: 1 hour 30 minutes

Total number of marks: 75

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\, dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}\, dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

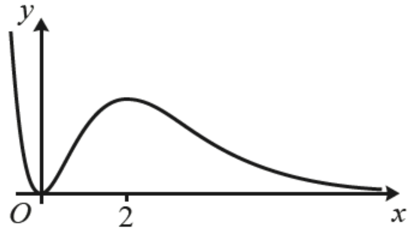
$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

1.



The diagram shows part of the curve $y = x^2e^{-x}$.

(a) Use the trapezium rule with 4 intervals of equal width to find an estimate for $\int_0^2 x^2e^{-x}\,dx$. Give your answer correct to **3** significant figures. **[4]**

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for $\int_0^2 x^2e^{-x}\,dx$. **[1]**

(c) Explain why it is not clear from the diagram whether the value from part **(a)** is an under-estimate or an over-estimate for $\int_0^2 x^2e^{-x}\,dx$. **[2]**

2.

$$p = \log_a 16$$

$$q = \log_a 25$$

where a is a constant.

Find in terms of p and/or q,

(a) $\log_a 256$

(1)

(b) $\log_a 100$

(2)

(c) $\log_a 80 \times \log_a 3.2$

(2)

3.

A curve has equation

$$y^3e^{2x} + 2y - 16x = k$$

where k is a constant.

The curve has a stationary point on the y-axis.

Determine the value of k

[7 marks]

4.

A singer has a social media account with a number of followers. The singer releases a new song and the number of followers grows exponentially.

The number of followers, F, may be modelled by the formula

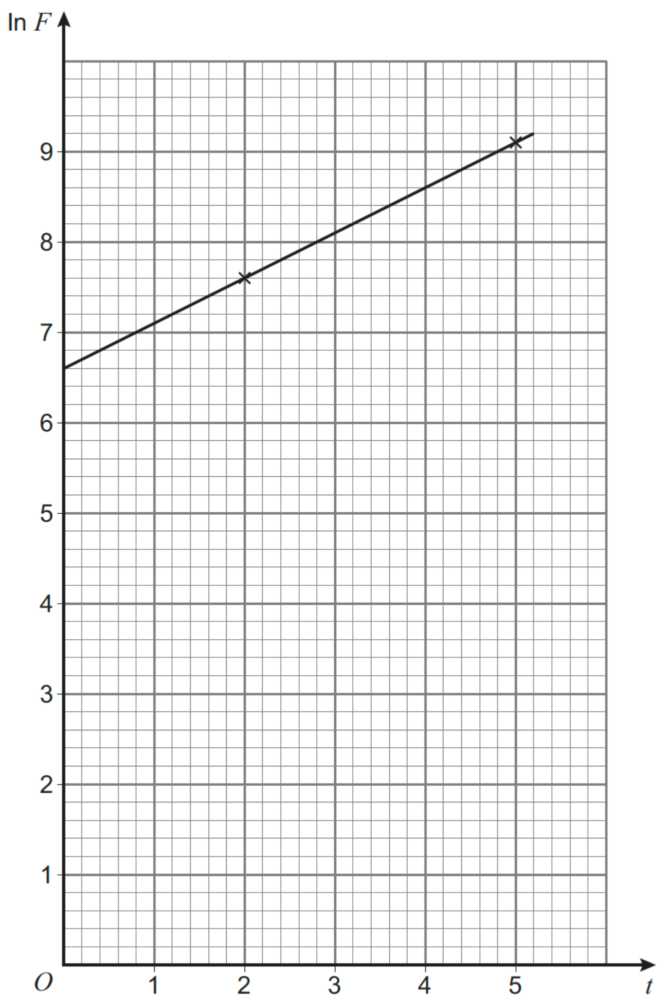
$$F = a\mathrm{e}^{kt}$$

where t is the number of days since the song was released and a and k are constants.

- Two days after the song is released the account has 2050 followers.
- Five days after the song is released the account has 9200 followers.

On the graph below ln F has been plotted against t for these two pieces of data.

A line has been drawn passing through these two data points.



(a) (i) Show that $\ln F = \ln a + kt$

[2 marks]

(a) (ii) Using the graph, estimate the value of the constant a and the value of the constant k

[4 marks]

5.

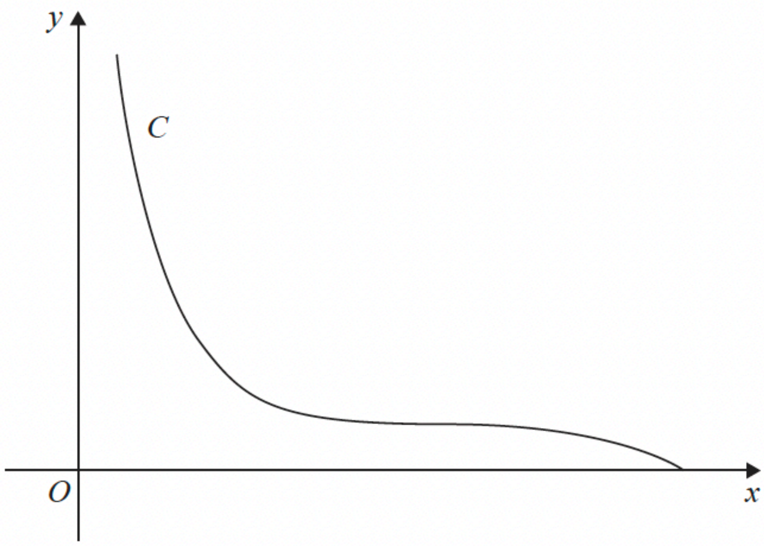


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = (t+3)^2 \qquad y = 1 - t^3 \qquad -2 \leqslant t \leqslant 1$$

The point P with coordinates (4, 2) lies on C.

(a) Using parametric differentiation, show that the tangent to C at P has equation

$$3x + 4y = 20$$

(5)

The curve C is used to model the profile of a slide at a water park.

Units are in metres, with y being the height of the slide above water level.

(b) Find, according to the model, the greatest height of the slide above water level.

(1)

6.

(a) Use a suitable substitution to show that

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x$$

can be written as

$$\frac{1}{2}\int_a^9 \left(2u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$$

where a is a constant to be found.

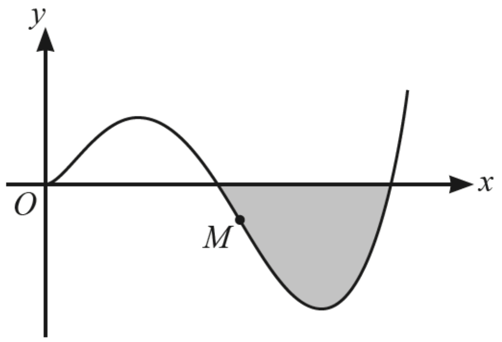
[5 marks]

(b) Hence, or otherwise, show that

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x = \frac{1322}{15}$$

[4 marks]

7.



The diagram shows the curve with equation $y = (x^3 - 2x^2)\ln x$. The curve has a point of inflection at the point M.

(a) **(i)** Show that the x-coordinate of M satisfies the equation

$$x = \frac{6 + (4 - 6x)\ln x}{5}.$$ **[5]**

(ii) Use an iterative formula, based on the equation in part **(a)(i)**, to determine the x-coordinate of M correct to **2** decimal places. Use an initial value of 1.1 and show the result of each step of the iterative process. **[2]**

(b) Determine the exact area of the shaded region, giving your answer in the form $p\ln q - r$, where p and r are positive rational numbers and q is a positive integer. **[6]**

8.

A scientist is monitoring the decline in the population of a certain endangered species of animal in an area where their natural habitat has been damaged.

As a model, the scientist proposes that the rate of decline per year of the population is given by $\frac{1}{80}P^2$, where P is the size of the population t years after the start of the modelling.

(a) Explain how this model gives rise to the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{1}{80}P^2.$$ **[1]**

The scientist notes that at the start of the monitoring the population is 120.

(b) Use the model to determine an expression for P in terms of t. **[4]**

(c) Use the model to determine the time it takes for the population to reach 10. **[2]**

9.

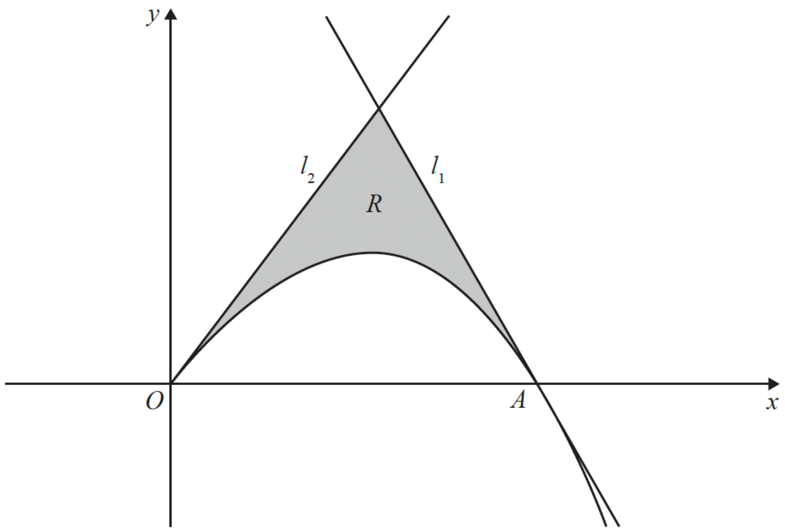


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the x-axis at the point A.

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A.

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R, shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R.

(5)

Mechanics

10.

A particle P is moving with constant acceleration $(-5\mathbf{i}+2\mathbf{j})\,\text{m s}^{-2}$. At time $t = 0$ seconds, P is at the origin and has velocity $(\mathbf{i}+3\mathbf{j})\,\text{m s}^{-1}$.

(a) Find, in terms of **i** and **j**, the displacement of P at time $t = 2$ seconds. **[2]**

(b) Determine the speed of P at time $t = 2$ seconds. **[4]**