U6 Ma Mock Teacher X 24-25 SOLUTIONS [75]

1.

(a)		$0.5\times0.5\ \{0 + 4e^{-2} + 2\left(0.25e^{-0.5} + e^{-1} + 2.25e^{-1.5}\right)\}$	**B1**	**1.1a**	State the 4 correct non-zero y-values and no others	Exact values (including unsimplified) or decimal equivs (0, 0.1516, 0.3679, 0.5020, 0.5413), which could be truncated or rounded For the first value, if $0e^0 = 1$ is seen then allow credit for the unsimplified value; if however it is only ever seen as 1 then this is **B0** but **M1M1** could still be awarded **B0** if other ordinates seen, unless clearly not intended to be used
			M1*	**1.1a**	Attempt to find area between $x = 0$ and $x = 2$, using $k\{y_0 + y_n + 2(y_1 + \ldots + y_{n-1})\}$	Big brackets need to be seen or implied Attempts at y-values must be correctly placed (but no need to see $y = 0$ explicitly) If no earlier evidence of y-values seen (eg in a table) then allow **M1**for the correct structure with 4 of the 5 values being correct Condone using more than 4 intervals as long as values equally spaced between $x = 0$ and $x = 2$
			M1d*	**1.1a**	Use $k = 0.5 \times 0.5$ soi	Dep on previous **M1** Or using $k = 0.5h$, with h consistent with their different number of intervals
		= 0.646	**A1**	**1.1**	Obtain 0.646	Allow answers > 3sf, as long as they round to 0.646 **A0** if not using 4 strips, even if 0.646 is obtained No credit if no evidence of using the trapezium rule shown
			[4]			Using separate strips (a triangle and then trapezia) is an acceptable method, and marks should be awarded as per the main MS (ie y-values / structure / widths / final answer)
(b)		Use more trapezia, of a lesser width, over the same interval	**B1** **[1]**	**2.4**	Convincing reason	Allow just 'more trapezia' or 'narrower trapezia' Could refer to strips or intervals
(c)		E.g. There is a point of inflection within the given range…	**B1**	**2.4**	Curve is **both** convex and concave	**Comment about the shape** Referring to increasing and decreasing gradients is correct, but increasing and decreasing curve is not Allow **BOD** if muddles about which part of the curve is convex and which is concave
		… so the trapezia initially over-estimate but then under-estimate	**B1** **[2]**	**2.2a**	The tops of trapezia are **both** above and below the curve	**Comment about the estimates** If candidates refer to 'it' rather than 'trapezia' then allow **BOD** **B** marks are independent **See appendix for further examples**

2.

9(a)	$2p$	B1	1.1b
		(1)	
(b)	$\log_a 100 = \log_a 4 + \log_a 25$	M1	1.2
	$\log_a 16^{\frac{1}{2}} + \log_a 25 = \frac{1}{2}p + q$	A1	1.1b
		(2)	
(c)	e.g. $\log_a 80 \times \log_a 3.2 = \left(\log_a 16 + \log_a 5\right)\times\left(\log_a 16 - \log_a 5\right)$	M1	3.1a
	$\left(p + \frac{1}{2}q\right)\times\left(p - \frac{1}{2}q\right)$ or $p^2 - \frac{1}{4}q^2$	A1	1.1b
		(2)	
			(5 marks)

3.

Uses implicit differentiation, with $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2\frac{\mathrm{d}y}{\mathrm{d}x}$ seen.	3.1a	M1	$y^3e^{2x}+2y-16x=k$ $3y^2e^{2x}\frac{\mathrm{d}y}{\mathrm{d}x}+2y^3e^{2x}+2\frac{\mathrm{d}y}{\mathrm{d}x}-16=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0, x=0$ $2y^3-16=0$ $y=2$ $2^3e^0+4-16\times0=k$ $k=12$
Uses product rule to differentiate y^3e^x and obtains $Ay^2e^{2x}\frac{\mathrm{d}y}{\mathrm{d}x}+By^3e^{2x}$	3.1a	M1	
Obtains correctly $3y^2e^{2x}\frac{\mathrm{d}y}{\mathrm{d}x}+2y^3e^{2x}+2\frac{\mathrm{d}y}{\mathrm{d}x}-16=0$	1.1b	A1	
Substitutes $\frac{\mathrm{d}y}{\mathrm{d}x}=0, x=0$ into their differentiated equation or rearranged equation to obtain a value for y. Their equation needs to have contained either $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2\frac{\mathrm{d}y}{\mathrm{d}x}$ and involve e^{2x}	3.1a	M1	
Obtains y = 2 Must have achieved M1M1A1M1 so far. PI substituting $y=\frac{2}{e^{\frac{2x}{3}}}$ and x=0 into $y^3e^{2x}+2y-16x$	1.1b	A1	
Substitutes $x=0$ and their y = 2 into $y^3e^{2x}+2y-16x$ to obtain a value of k	3.1a	M1	
Deduces k =12 Must have achieved all previous marks	2.2a	R1	
Question 19 Total		**7**	

4.

10(a)(i)	Takes logarithms of both sides PI	1.1a	M1	$F = ae^{kt}$ $\ln F = \ln ae^{kt}$ $\ln F = \ln a + \ln e^{kt}$ $\ln F = \ln a + kt$
	Derives the required equation Must see either: $\ln F = \ln a + \ln e^{kt}$ Or $\ln F = \ln a + kt\ln e$	2.1	R1	
	Subtotal		**2**	

10(a)(ii)	Equates $\ln a$ to intercept value $6.55 \le \ln a \le 6.65$ Or Uses one/two points on graph to form 1/2 equations in $\ln a$ and k	3.1b	M1	$\ln a = 6.6$ $a = 735$ gradient $= 0.5 = k$
	Equates k to a gradient value Or Solves equation(s) or uses F to find a value of a and/or a value of k	1.1a	M1	
	Obtains a value for a AWFW 700 to 773	1.1b	A1	
	Obtains a value for k AWFW 0.45 to 0.55	1.1b	A1	
	Subtotal		**4**	

5.

10(a)	$x = 4, y = 2 \Rightarrow t = -1$	B1	2.2a
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -3t^2 \times \frac{1}{2(t+3)}$	M1	1.1b
	$\frac{dy}{dx} = -3(-1)^2 \times \frac{1}{2(-1+3)} = -\frac{3}{4}$	M1	1.1b
	$\Rightarrow y - 2 = -\frac{3}{4}(x-4)$ or $\Rightarrow y = -\frac{3}{4}x + c \to 2 = -\frac{3}{4} \times 4 + c \Rightarrow c...$	ddM1	2.1
	$y - 2 = -\frac{3}{4}(x-4) \Rightarrow 4y - 8 = -3x + 12$ or $c = 5 \Rightarrow y = -\frac{3}{4}x + 5$ $\Rightarrow 3x + 4y = 20$*	A1*	1.1b
		(5)	
(b)	Maximum height is 9m	B1	3.4
		(1)	
			(6 marks)

6.

18(a)	Selects the substitution $u=2x+1$ and differentiates or uses it to replace $2x+1$ in the integrand.	3.1a	B1	Let $u=2x+1$ $\frac{du}{dx}=2 \Rightarrow dx=\frac{1}{2}du$ $4x+1=2u-1$ $\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx$ $=\int_1^9 (2u-1)(u)^{\frac{1}{2}}\frac{1}{2}du$ $=\frac{1}{2}\int_1^9 \left(2u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$
	Differentiates their substitution and uses the result to replace dx in the integral.	1.1a	M1	
	Makes a complete substitution to write the integrand in terms of u leading to an integrand of the form $A(2u-k)u^{\frac{1}{2}}$ Or FT their substitution $u=(2x+1)^{\frac{1}{2}}$ or $u^2=2x+1$ leading to an integrand of the form $A\left(2u^2-1\right)u^2$	3.1a	M1	
	Obtains correct lower limit for their substitution	1.1a	M1	
	Completes a reasoned argument to show the required result with $a=1$	2.1	R1	
	Subtotal		**5**	
18(b)	Integrates to obtain $\frac{1}{2}\frac{4u^{\frac{5}{2}}}{5}$ or $\frac{4u^{\frac{5}{2}}}{5}$ or $-\frac{1}{2}\frac{2u^{\frac{3}{2}}}{3}$ or $\frac{2u^{\frac{3}{2}}}{3}$	1.1a	M1	$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx$ $=\frac{1}{2}\int_1^9 \left(2u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$ $=\frac{1}{2}\left[\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}\right]_1^9$ $=\frac{1}{2}\left[\left(\frac{4\times 9^{\frac{5}{2}}}{5}-\frac{2\times 9^{\frac{3}{2}}}{3}\right)-\left(\frac{4\times 1^{\frac{5}{2}}}{5}-\frac{2\times 1^{\frac{3}{2}}}{3}\right)\right]$ $=\frac{1322}{15}$
	Obtains $\frac{1}{2}\left(\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}\right)$ or $\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}$	1.1b	A1	
	Substitutes limits explicitly into their integrated expression of the form $Au^{\frac{5}{2}}-Bu^{\frac{3}{2}}$ Where A and B are both positive FT their non-zero a Condone omission of powers on substitution of 1	1.1a	M1	
	Completes argument to show the given result with no unrecovered slips.	2.1	R1	
	Subtotal		**4**	

7.

(a)	(i)	$y=\left(x^3-2x^2\right)\ln x \Rightarrow \frac{dy}{dx}=...$	M1	2.1	**M1** for attempt to differentiate using the product rule (oe) – answer must be of the form $\left(x^3-2x^2\right)\times\frac{k_1}{x}+\left(k_2x^2+k_3x\right)\ln x$ for non-zero constants k_1,k_2,k_3	Condone invisible brackets for this mark
		$\frac{dy}{dx}=\left(x^3-2x^2\right)\left(\frac{1}{x}\right)+\left(3x^2-4x\right)\ln x$	A1	1.1	**A1** for a correct first derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
		$\frac{d^2y}{dx^2}=2x-2+\frac{3x^2-4x}{x}+(6x-4)\ln x$	A1	1.1	**A1** for a correct second derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
		$\frac{d^2y}{dx^2}=0\Rightarrow 2x-2+\frac{3x^2-4x}{x}+(6x-4)\ln x=0$	M1	1.1	Setting the second derivative (which if simplified would be of the form $ax+b+(cx+d)\ln x$ with non-zero constants a, b, c and d) equal to zero	
		$5x-6=(4-6x)\ln x\Rightarrow x=\frac{6+(4-6x)\ln x}{5}$	A1 **[5]**	2.2a	**AG** – so sufficient working must be shown – at least one intermediate line of working from second derivative set equal to zero to given answer	**Any** errors seen (e.g. any missing/invisible brackets) is **A0**
(a)	(ii)	$x_{n+1}=\frac{6+(4-6x_n)\ln x_n}{5}$ $x_1=1.1$ $x_2=1.150438...$ $x_3=1.118643...$ $x_4=1.139191...$ $x_5=1.126105...$ $x_6=1.134521...$	B1	1.1	Uses given result and given starting value (of 1.1) to obtain correct x_2 and x_3 (so first two iterations after the initial value of 1.1) to at least 2 dp (rot) – but all stated values in these two terms must be correct	
		x-coordinate of M is 1.13	B1 **[2]**	2.2a	Must be stated to **2** dp only – **not dependent on the first B mark** – can be awarded if either of x_2 and x_3 are incorrect (assume that the iterative process corrected itself or a slip in the candidate writing down an earlier value)	Must be clear that x is 1.13 (and not the final term shown in the iterative process e.g. $x_6=1.13$ only is **B0**) – this mark can be awarded from using alternative iterative methods e.g. Newton-Raphson

(b)		Curve crosses the x-axis at 1 and 2	B1*	3.1a	Correct x-intercepts (soi) – ignore mention of $x = 0$	Could be seen as limits on integral(s)
		$\int\left(x^3-2x^2\right)\ln x\,\mathrm{d}x = \ldots$	M1*	2.1	**M1** for attempt at integration by parts – **must be of the form** $(ax^4+bx^3)\ln x \pm \int(cx^4+dx^3)\times\frac{1}{x}(\mathrm{d}x)$ for non-zero constants a, b, c and d	Limits not required for this and the next two **A** marks (so condone incorrect limits too for these 3 marks)
		$=\left(\frac{1}{4}x^4-\frac{2}{3}x^3\right)\ln x-\int\left(\frac{1}{4}x^4-\frac{2}{3}x^3\right)\left(\frac{1}{x}\right)\mathrm{d}x$	A1	1.1	correct first application (allow un-simplified)	dx not required and integral sign(s) can be implied
		$=\left(\frac{x^4}{4}-\frac{2x^3}{3}\right)\ln x-\frac{x^4}{16}+\frac{2x^3}{9}(+c)$	A1	1.1	cao (allow un-simplified)	
		$\left\{\left(\frac{16}{4}-\frac{16}{3}\right)\ln 2-1+\frac{16}{9}\right\}-\left\{0-\frac{1}{16}+\frac{2}{9}\right\}$	M1dep*	1.1	Uses correct limits completely correctly $\pm(F(2)-F(1))$ in **their** fully integrated expression – need not be simplified (or equivalent)	Do not condone invisible brackets unless recovered
		$\int_1^2\left(x^3-2x^2\right)\ln x\,\mathrm{d}x=-\frac{4}{3}\ln 2+\frac{89}{144}$ $\Rightarrow$ Area $=\frac{4}{3}\ln 2-\frac{89}{144}$	A1	3.2a	Must be of this form but allow exact equivalents (for example, $\frac{1}{3}\ln 16-\frac{89}{144}$) but the p and r must be positive rational numbers and q must be a positive integer **Be aware of those who consider** $\int_1^2(2x^2-x^3)\ln x\,\mathrm{d}x$ **which is correct**	For reference – one possibility is: $p=\frac{4}{3}, q=2, r=\frac{89}{144}$ **These values do not need to be stated explicitly**
			[6]			

8.

(a)		(The rate of change with respect to time is $\frac{\mathrm{d}P}{\mathrm{d}t}$), which is $-\frac{1}{80}P^2$ because the population is in **decline**.	B1 [1]	3.3	Must see use of **decrease/decline** linked to the negative sign. Condone answers that do not refer to $\frac{\mathrm{d}P}{\mathrm{d}t}$ Examples: • "it's $-\frac{1}{80}P^2$ because decreasing" B1 • "decreasing" [and nothing further] B0 • "decline implies negative sign" B1
(b)		$\frac{\mathrm{d}P}{P^2}=-\frac{\mathrm{d}t}{80}$ oe	M1*	3.4	Attempt to separate variables, must see P and dP on the same side May see $\frac{\mathrm{d}t}{\mathrm{d}P}=\frac{-80}{P^2}$ as a first step. In this case M1 is implied by a subsequent attempt to integrate the RHS w.r.t. P (so do not give M1 for this statement alone).
		$-\frac{1}{P}=-\frac{t}{80}(+c)$ oe	A1	1.1	May see $\frac{1}{P}=\frac{t}{80}(+c)$ etc. Allow without $+c$
		$c=-\frac{1}{120}$ oe $\left(\frac{1}{P}=\frac{3t+2}{240}\right)$	M1 dep*	2.1	Substitute (0, 120) and attempt to find c (which may not be this value). Must reach a value of c for this mark.
		$P=\frac{240}{3t+2}$ oe	A1 [4]	1.1	Must be in terms of P (P=…) but need not be simplified. isw incorrect attempts to simplify following P=…
(c)		$10=\frac{240}{3t+2}$ or $t=\frac{80}{P}-\frac{2}{3}$	M1	3.4	Either substitute $P = 10$ into their equation from (b) or rearrange to make t the subject.
		$t=7\frac{1}{3}$ (years) oe	A1 [2]	1.1	Accept 7.33 (3sf), ignore units.

9.

10(a)	$8(4)-4^{\frac{5}{2}}=32-32=0$	B1	1.1b
		(1)	
(b)	$8-\frac{5}{2}x^{\frac{3}{2}}$	B1	1.1b
	$x=4\Rightarrow\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}8-\frac{5}{2}\times8=-12$ $\Rightarrow y\{-0\}="-12"(x-4)$	M1	1.1b
	$12x+y=48$ *	A1*	1.1b
		(3)	
(c)	Attempts to find one of the coordinates of the point of intersection $y=8x,\ 12x+y=48\Rightarrow y=19.2$ (or $x=2.4$)	M1	1.1b
	Triangle area is $\frac{1}{2}\times4\times"19.2"\left(=38.4\text{ or }\frac{192}{5}\right)$ or $\int_0^{"2.4"}8x\ \mathrm{d}x+\int_{"2.4"}^{4}"(48-12x)"\ \mathrm{d}x$	dM1	3.1a
	$\int\left(8x-x^{\frac{5}{2}}\right)\mathrm{d}x=4x^2-\frac{2}{7}x^{\frac{7}{2}}$	B1	1.1b
	$A=38.4-\left["4x^2-\frac{2}{7}x^{\frac{7}{2}}"\right]_0^4=38.4-64+\frac{256}{7}$	ddM1	3.1a
	$=\frac{384}{35}$	A1	1.1b
		(5)	
			(9 marks)

10.

(a)		$[\mathbf{s}=]\,2(\mathbf{i}+3\mathbf{j})+0.5\times2^2\times(-5\mathbf{i}+2\mathbf{j})$ or $2\begin{pmatrix}1\\3\end{pmatrix}+0.5\times2^2\times\begin{pmatrix}-5\\2\end{pmatrix}$	**M1**	**3.3**	Apply $\mathbf{s}=\mathbf{u}t+0.5\mathbf{a}t^2$ correctly with correct values of **u**, **a** and t – if using integration then for this mark we must see the correct expression $\begin{pmatrix}1\\3\end{pmatrix}t+\frac{1}{2}\times\begin{pmatrix}-5\\2\end{pmatrix}t^2$ **with** $t=2$ subst.	
		$[\mathbf{s}=]-8\mathbf{i}+10\mathbf{j}$ (m)	**A1** **[2]**	**1.1**	or $\begin{pmatrix}-8\\10\end{pmatrix}$	ISW if correct vector converted to scalar
(b)		$\mathbf{v}=(\mathbf{i}+3\mathbf{j})+2(-5\mathbf{i}+2\mathbf{j})$	**M1***	**3.3**	Apply $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ with correct values of **u**, **a** and t (or other complete method to find **v**)	Allow from integration but must have correct expression for **v** with $t=2$ substituted
		$\mathbf{v}=-9\mathbf{i}+7\mathbf{j}$	**A1**	**1.1**	or as a column vector (possibly implied by correct magnitude)	
		$\lvert\mathbf{v}\rvert=\sqrt{(-9)^2+7^2}$	**M1dep***	**3.4**	Correct method for the speed of P at time $t=2$ – condone $\sqrt{-9^2+7^2}=\sqrt{\pm81+49}$	
		$\lvert\mathbf{v}\rvert=11.4$ (m s^{-1})	**A1** **[4]**	**1.1**	Allow $\sqrt{130}$ or awrt 11.4 www – must follow from correct $\mathbf{v}=-9\mathbf{i}+7\mathbf{j}$ (so **M1 A0 M1 A1** is not possible)	11.4017542…